Slide 1

Administrivia

- Reminder: Homework 6 code due today. Due dates for Homework 7 on Web (after holiday).
- Reminder: Quiz 5 Tuesday. Likely topic is GUIs.

Slide 2

Trees — Mathematical Definition

- One definition —
  - Set of nodes, one called root.
  - Set of edges (directed connections between nodes).
  - Root has no incoming edges; all other nodes have exactly one (from parent).
  - Each node can have 0 or more outgoing edges (to children — if none, leaf node).
- Another, recursive definition — tree is one node connected by edges to 0 or more subtrees.
- This is a general tree — e.g., to represent hierarchy such as filesystem.

Slide 3

Implementing Trees

- Define `Node` data structure, analogous to linked list, with reference to data and references to children (array or linked list or . . .).
- Easier if number of children is limited to two, and this turns out to be sufficiently useful in practice — "binary tree". Then `Node` consists of pointers to data and left and right subtrees.

Slide 4

Tree Traversals

- For linked lists we defined a way to visit all elements — "iterator". Is there something analogous for trees?
- Well — three orders that are easy to define and implement:
  - Preorder — root first.
  - Postorder — root last.
  - Inorder — leftmost subtree first, then root, then remaining subtrees. (Admittedly a little weird for non-binary trees.)
- (Sketch some code for at least one of these.)

Slide 5

Sorted Binary Trees (Binary Search Trees)

- Key property — everything in the left subtree is smaller than the root, and everything in the right is bigger.
- Why is this useful? If you want a data structure to hold a collection that will be searched frequently, what are the choices? and how fast is each to search? to modify (insert/remove)? Compare approximate times for arrays (sorted and unsorted), linked lists (sorted and unsorted), sorted binary tree.
- (Sketch some code for `add` and `find`. `remove` is trickier . . .)

Slide 6

Priority Queues, Revisited

- Several data structures we could use to implement priority queue ADT:
  - Unsorted linked list.
  - Sorted linked list.
  - Sorted binary tree.

  Compare how much work to add/remove if N elements. Can we do better? Maybe!

Slide 7

Heaps

- Heap is another tree-based data structure, with two properties:
  - A node is always “bigger than” both its children.
  - Tree is “complete”.
- For a priority queue, we want to retrieve the “biggest” thing (for game problem, smallest update time). Does this seem useful?
- Note also that we can store a complete binary tree in an array.
- How to insert and remove? Compare running times.

Slide 8

Minute Essay

- Sketch what a sorted binary tree of integers would look like after adding the following:

  5, 4, -1, 10, 6, 20.
- Now sketch what a heap of integers (ordered to put smallest values at the top) would look like after adding the same values.

Slide 9

Minute Essay Answer

- The BST:

```
     5
    / \
   4   10
  /   /  \
-1   6    20
```

- The heap:

```
     -1
    /    \
   5      4
  / \    /
10   6  20
```