Slide 1

Administrivia

- First homework on Web by Monday, due following Monday.

Slide 2

Minute Essay From Last Lecture

- Question: What have you liked/disliked about previous math courses?
- Answers indicate that people like/dislike different things! e.g., some liked calculus and some hated it.

  Notice that this is “discrete math” as opposed to the kind of “continuous” math involved in calculus.

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Why Study Propositional Logic?

- Because it's conceptually related to Boolean algebra (used in programming, circuit design, etc.).
- Because it's related to proofs, which you should know a bit about.
- As an example of a "formal system" — represent something symbolically, define and apply rules for manipulating symbols, etc. Other examples in automata theory, theory of databases, etc.
- Because when you ask Dr. Theory (Myers) what you should learn in this course, he says "logic, logic, logic, logic!"
- Because after logic, the rest of the course will (probably) seem easy!

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Propositional Logic — The Big Picture

- Underlying many fields is a notion of "valid argument", one thing "following logically" from another — math, science, law, etc. (Consider example at the start of chapter 1.)
- Can define precisely what this means using natural language, but it's difficult and clumsy.
- If we use mathematical notation instead, it's easier to produce/follow chains of reasoning.

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Statements / Propositions

- Definition — something (in natural language, a sentence) that is either true or false. (We might not know which.)
- Which of these are statements?
  - Water is wet.
  - Water is not wet.
  - Is the sky blue on Venus?
  - There is life on Mars.
- Notational convention — A, B, C, etc., are statements.

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Connectives

- Can build up more complicated statements by combining simpler ones, using "connectives" — each has intuitive meaning, formal definition.
- "and" connective pretty clear — $A \wedge B$ defined by truth table
- "or" connective also pretty clear — $A \vee B$ defined by truth table

  Notice that this is "inclusive or" — not always the same as what we mean by "or" in natural language.
- "not" connective also pretty clear — A' defined by truth table
- "implies" connective is trickier — $A \rightarrow B$ defined by truth table

  Why define it that way? Stay tuned . . .
- "is equivalent to" connective also pretty clear — $A \leftrightarrow B$ defined by truth table

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Why Did We Define Implication That Way?

- Definition of $A \rightarrow B$ when A is true seems reasonable, right?
- When A is false, though — why say $A \rightarrow B$ is true?
  - "Benefit of the doubt" argument: We have to call it either true or false, and it's not obviously false.
  - "It's math" argument: Maybe this definition *doesn't* express some fundamental truth. In some sense math is its own universe, and we can define things any way we want (though we hope the definitions fit together in a nice way and maybe have applications).
    (In fact, some treatments of propositional logic just define implication formally, in terms of other connectives, and don't try to justify it.)

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Compound Statements / Well-Formed Formulas

- We can "nest" connectives, e.g., $(A \wedge B)'$.
- Natural-language equivalents:
  - Water is wet and grass is green.
  - If Jo(e) is a CS major, Jo(e) must take this course.
- We can define a notion of "well-formed formula" (wff) based on this (formal definition should be recursive, and we'll do that later) — basically, a "sensible" combination of statement letters, connectives, and parentheses.
- Notational convention — P, Q, . . . for wffs.
- We can use truth tables to figure out truth values for wffs. (How many rows do we need?) Let's do an example . . .

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Compound Statements, Continued

- As an example, let's try turning the example at the start of chapter 1 into formulas, using the following:

  A is "The client is guilty."

  B is "The knife was in the drawer."

  C is "Jason P. saw the knife."

  D is "The knife was there on Oct. 10."

  E is "The hammer was in the barn."

- And then we hope we can somehow use the formulas to help us decide whether the conclusion follows from the premises.

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More Definitions

- Some wffs are always true — "tautologies". Examples?
- Some wffs are always false — "contradictions". Examples?
- We can talk about two wffs P and Q being "equivalent" — $P \leftrightarrow Q$ is a tautology.

  Write $P \Leftrightarrow Q$.

  Table of common equivalences on p. 8.

  Additional widely-used equivalences — "De Morgan's Laws".

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Propositional Formulas in Other Contexts

- Notice similarities between the connectives here and
  - Boolean expressions in programming languages.
  - Expressions for “advanced search” in some search engines, database queries, etc.
- Can use rules we have so far to simplify such expressions.

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Valid Arguments

- Now we want to capture notion of “valid argument” — formal version of what someone familiar with proofs would recognize as such.
- Idea is that we have “hypotheses” $P_1, P_2, \ldots, P_n$ and “conclusion” Q, and we want to know when we can be sure that the truth of the hypotheses guarantees the truth of the conclusion — i.e., when is

$$(P_1 \land \ldots \land P_n) \rightarrow Q$$

  a tautology?
- Could we use truth tables? If we can, would we always want to?

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Valid Arguments, Continued

- A more algorithmic view — apply "derivation rules" to construct a "proof sequence".

  Idea is that we have a list of wffs that we know are true any time all the hypotheses ($P_1, P_2, \ldots P_n$) are true. Then we proceed thus:

  1. Initialize this list to include just $P_1, P_2, \ldots P_n$.
  2. If conclusion Q is on the list, stop.
  3. Apply a derivation rule to one or more wffs in the list, producing a new wff X. Add X to the list.
  4. Go to step 2.

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Derivation Rules

- What kind of "derivation rules" would be good?
  - When we apply one, we want it to be the case that if the wffs we start with are true, the wff we derive is also true — system is "sound". "Everything we can prove is true."
  - Together they are powerful enough to allow us to construct proof sequences for all true statements — system is "complete". "Everything that is true has a proof." (Possible here, but not for more complicated kinds of logic!)

Derivation Rules, Continued

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- Two groups of basic rules:
  - Equivalence rules (two-way) — p. 23.
  - Inference rules (one-way) — p. 24.

  ("Do I have to memorize these?" No. Exams and quizzes will be open book.)
- Formally, these rules are true because we can prove them using truth tables.
- They should also seem plausible, maybe even "obviously true".
- Can derive additional rules; table on p. 31 lists some.

Proof Sequences

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- So, let's do an example:
  - Hypotheses:

    A

    B
  - Conclusion:

    $(A \wedge B) \vee C$
- "Justifications" we write down for each step aren't technically required for a valid proof sequence. We put them in to help human readers.

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Proof Sequences, Continued

- Two things involved in constructing proofs:
  - Applying the rules correctly — not so difficult, if you correctly match up your formula with the rule.
  - Knowing which rule to apply — more difficult, gets a little easier with practice. Also see hints on p. 26.
- Time permitting, let's do the lawyer example. (*We'll do this next time.*)

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So, What Does This Buy Us?

- Yes, this can seem long and tedious. But . . .
- It's in some ways easier than other approaches, and certainly more reliable.
- Compare to "word problems" in algebra — first convert from natural language to math, apply math, convert back — with practice, easier and more reliable than guessing.
- In a way, we're replacing thinking with symbol manipulation!

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Minute Essay

- Suppose we have

  W is “Water is wet.”

  L is “There is life on Mars.”

- Write as wffs the following:

  “Water is wet and there is life on Mars.”

  “There is life on Mars if water is wet.”