Slide 1

## Administrivia

- None.

Slide 2

## Exhaustive Proof / Proof By Cases

- Idea here is to prove by considering each "case" separately. Only works if there are finitely many. (Recall result from propositional logic that allows this.)
- Simple example: To show that for all integers $x$ with $0 \le x \le 4$, $x^2 < 20$, five cases to consider.
- Slightly more complex example: To show something for all integers, can consider two cases, odd integers and even integers. (Aside: How shall we define "even"? Is zero even?)
- Much more complex example: Computer-assisted proof of 4-color map theorem (1976, used almost 2000 separate cases).

## Direct Proof

- Idea here is to show $P \rightarrow Q$ like we've been doing — assume $P$ and derive $Q$ — but less formally.
- Example: Show that for integers $p$ and $m$, if $p$ is even and $m$ is positive, $p^m$ is even.

**Slide 3**

## Proof by Contraposition

- Idea is based on a derived rule from propositional logic: If $Q' \rightarrow P'$, then $P \rightarrow Q$.

  So if proving $P \rightarrow Q$ is difficult, we can try proving $Q' \rightarrow P'$ instead.
- Example: Show that if $m$ and $n$ are integers and $m + n$ is even, either $m$ and $n$ are both even or $m$ and $n$ are both odd.

**Slide 4**

Slide 5

## Proof By Contradiction

- Idea is based on another rule we could prove using propositional logic: If $(P \wedge Q') \rightarrow false$, then $P \rightarrow Q$.

  So if proving $P \rightarrow Q$ is difficult, we can try assuming $P \wedge Q'$ and "deriving a contradiction".

  Note that sometimes $P$ is just $true$.

- Example: Show that $\sqrt{2}$ is irrational.

Slide 6

## Minute Essay

- Find a counterexample for the following conjecture: "If $x$ is an integer, $\sqrt{x}$ is an integer."
- To show that there is no largest prime, we could assume $P$ and derive a contradiction. What is $P$? (You don't have to show there's no largest prime, just say what $P$ is.)
- (Reminder: Homework 2 due.)