Slide 1

Administrivia

- None.

Slide 2

Minute Essay From Last Lecture

- Question: Consider the following recursive definition of a sequence:

$$\begin{aligned} S(1) &= 1 \\ S(n) &= 10S(n-1) + 1, \quad \text{for } n > 1 \end{aligned}$$

What are $S(1), S(2), \ldots S(5)$?

- Answer?

Slide 3

Recurrence Relations

- Recall the silly example of defining a sequence recursively:

$$\begin{aligned} S(1) &= 1 \\ S(n) &= S(n-1) \times 10, \quad \text{for } n > 1 \end{aligned}$$

  Expanding out some terms, it seems fairly obvious that an equivalent definition would be $S(n) = 10^{n-1}$ — a "closed-form solution" to the recurrence relation given in the second line of the definition.

- We'll look at various ways to come up with such solutions — because they're generally easier to compute, but sometimes it will be much easier to write down the recursive definition.

Slide 4

Solving Recurrence Relations, Continued

- For the silly example

$$\begin{aligned} S(1) &= 1 \\ S(n) &= S(n-1) \times 10, \quad \text{for } n > 1 \end{aligned}$$

  we guessed a solution of $S(n) = 10^{n-1}$. Can we verify that this is the same as the recursive definition? yes, via a proof by induction ...

- Try another example — section 2.4 problem 75.

- Call this method "expand, guess, verify".

Slide 5

Solving Recurrence Relations, Continued

- Is there another way? In general, probably not, but there are some formulas for some frequently-occurring special cases.
- One is "first-order linear" recurrence relations. If

$$S(n) = cS(n-1) + g(n)$$

then we can show (see textbook for derivation) that

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n}(c^{n-i}g(i))$$

- Apply this to the two problems we did earlier — we should get the same results.

Slide 6

Minute Essay

- None — quiz.