Slide 1

Administrivia

- Midterm grades to be sent by e-mail this week or next.
- I plan to show up Friday with something to say about an interesting tangent. If you're here too, double attendance points.

Slide 2

Sets — Review

- Last week we reviewed/defined a lot of stuff about sets:
  - Ways to write/specify them.
  - Subsets and power sets.
  - Operations on sets.
  - Countability.
- Minute essay question: Suppose you have

  $A = \{2, 4, 6, 8\}$

  $B = \{1, 4, 9, 16\}$

  What are $A \cup B$, $A \cap B$, and $A - B$? How many elements are there in $\mathscr{P}(A)$?

Slide 3

Countable and Uncountable Sets, Recap

- We said last week that we'd say two sets are "the same size" if we could set up a one-to-one correspondence between them.
- We also said that we'd say a set S is countable if there's some way to write down all elements "in order" — i.e., set up a one-to-one correspondence with a subset of the positive integers. We showed that surprisingly many sets are countable — e.g., the rational numbers.
- So are all sets countable?

Slide 4

Countable and Uncountable Sets, Continued

- $\mathbb{R}$ is not countable. Proof is by contradiction. First we notice that we can set up a one-to-one correspondence between all real numbers and the real numbers greater than 0 and less than 1. Then we assume we can "list" those numbers and show that there's one we missed.
- We can also prove that S and $\mathscr{P}(S)$ are not "the same size", again by contradiction. (But not today.)
- (Is any of this crucially important to your understanding of computer science? Probably not, but it's too entertaining to skip.)

Slide 5

Counting (Combinatorics)

- "Counting" sounds too trivial for a college-level course, right? but consider situations in which you want to know how many things there are in a set but don't actually want to list them all:
  - Given what a password is supposed to look like (4 digits, 20 characters, etc.), how many are there? i.e., how easy would it be to guess?
  - Given a scheme for IP addresses, how many are possible? i.e., are there enough for everything we want to give one to?

Slide 6

Multiplication Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2, how many outcomes are there for the sequence "event 1, then event 2"?
- Pictorially, we could draw a tree, and then we can see there are $N_1 \times N_2$.
- This is easily extended by induction to sequences of more than two events.
- Example: If a password consists of 4 decimal digits, how many are there? (And if we allowed 10 seconds to try each one, how long would it take to try them all?)
- Example: If a license-plate number is 3 decimal digits followed by three alphabetic characters, how many are there?

Addition Principle

Slide 7

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2 (and the sets of “event 1 outcomes” and “event 2 outcomes” are disjoint), how many outcomes are there for the event “event 1 or event 2”?
- Fairly easy to see that there are $N_1 + N_2$ possibilities in all.
- This also is easily extended by induction to combinations of more than two events.
- Example: If you have to choose an elective from either the Department of Esoteric Pursuits (which offers 10 of them) or from the Department of Life Skills (which offers 6 of them), how many choices are there in all? (assuming no courses are cross-listed).

Combining the Addition and Multiplication Principles

Slide 8

- Example: How many phone numbers are there that have either area code 210 or area code 512?
- Example: How many 7-digit phone numbers have at least one repeated digit?

Decision Trees

- Sometimes it's useful to represent a sequence of choices as a "decision tree" and explicitly count leaf nodes.
- Example: How many ways are there to get 4 coin tosses with no sequences of three heads or three tails?

Slide 9

More Examples

- Section 3.2 problem 40.
- Section 3.2 problem 44.

Slide 10

Slide 11

Minute Essay

- If a password is at least 5 characters and no more than 8, where a character is either a digit or a lowercase character, how many possible passwords are there? (Okay to just write down an expression and not simplify — e.g., 10^4.)