Administrivia

- Homework 6 on Web. Due next Wednesday. Lots of problems, but many of them should be relatively quick and easy.

Slide 1

Minute Essay From Last Lecture

- Question: Suppose you select 6 marbles at random from a jar containing red, blue, yellow, and green marbles (at least 6 each). How many ways can this selection be made?
- Answer?

Slide 2

Slide 3

Probability — Equally-Likely Outcomes

- Basic definition: If S ("sample space") is a set of equally likely outcomes of some action (e.g., possible results of tossing a fair coin), and E ("event") is a subset of S, then we define the probability of E as

$$P(E) = \frac{|E|}{|S|}$$

Examples: Sequences of coin tosses, 5-card "hands" chosen from 52-card deck, etc.

- Note that $0 \leq P(E) \leq 1$. (Why?) When is $P(E) = 0$? When is $P(E) = 1$?
- Note that we can apply anything we know about sizes of sets. (E.g., if E_1 and E_2 are disjoint, what is $P(E_1 \cup E_2)$ in terms of $P(E_1)$ and $P(E_2)$?)

Slide 4

Probability — Not-Equally-Likely Outcomes

- One approach — extend previous definition by adding duplicates to sample space for outcomes that are more likely.
- Another approach — "probability distribution": For each x in sample space S, assign x a probability $p(x)$, such that

$$0 \leq p(x) \leq 1, \text{for all } x \in S$$

$$\sum_{x \in S} p(x) = 1$$

- Now for event E ($E \subseteq S$), we have

$$P(E) = \sum_{x \in E} p(x)$$

- Note that equally-likely-outcomes definition is a special case of the above.

Slide 5

Conditional Probability

- We could also consider, for two possibly related events E_1 and E_2, how likely it is that E_2 happens given that E_1 has happened — "conditional probability":

$$P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

  Intuitive idea is that here the "sample space" is limited to E_1.

- If it turns out that $P(E_2|E_1) = P(E_2)$, we call E_1 and E_2 "independent events". In this case, we can derive that $P(E_1 \cap E_2)$ is — what?

- Now we have the equivalent of our earlier addition and multiplication principles.

Slide 6

Minute Essay

- None — quiz.