Administrivia

- Reminder: Homework 6 due Wednesday.
- Reminder: Quiz 5 Wednesday.

Slide 1

Minute Essay From Last Lecture

- Question: If a fair coin is tossed four times, what's the probability of getting two heads and two tails given that there's at least one head and at least one tail?
- Answer?

Slide 2

Slide 3

Binary Relations, Review

- Formal definition: A binary relation ρ on a set S is a subset of $S \times S$. Usually this set is defined by some property of interest. For $a, b \in S$, we write $a \ \rho \ b$ iff (if and only if) (a, b) is in this subset.
- Examples:
  - $=$, $<$, $\leq$ on integers.
  - "Have the same parents", "have at least one parent in common" on people.
  - $\subseteq$ on sets.
  - Equivalence mod 10 on integers.

Slide 4

Properties of Binary Relations

- ρ is *reflexive* if $x \ \rho \ x$ for all $x \in S$.
- ρ is *symmetric* if $(x \ \rho \ y) \ \rightarrow \ (y \ \rho \ x)$ for all $x, y \in S$.
- ρ is *transitive* if $(x \ \rho \ y) \ \wedge \ (y \ \rho \ z) \ \rightarrow \ (x \ \rho \ z)$ for all $x, y, z \in S$.
- ρ is *antisymmetric* if $(x \ \rho \ y) \ \wedge \ (y \ \rho \ x) \ \rightarrow \ (x = y)$ for all $x, y \in S$.
- Can combine these in interesting ways . . .

Slide 5

Partial Ordering

- Idea: Generalize idea of "ordering" to include relations where not all pairs of elements can be ordered.
- Definition: ρ is a partial ordering if it's reflexive, antisymmetric, and transitive.
- Examples: $\leq$ on integers or reals, $\subseteq$ on sets.
- If finite, can represent with "Hasse diagram" (see examples in textbook).
- Related terms:
  - Successor, predecessor, immediate predecessor, immediate successor.
  - Least, greatest elements.
  - Minimal, maximal elements.

Slide 6

Equivalence Relation

- Idea: Generalize idea of "equals" to include relations where pairs of elements are equivalent but not identical.
- Definition: ρ is an equivalence relation if it's reflexive, symmetric, and transitive.
- Examples: $=$ on integers or reals, $(x \bmod n) = (y \bmod n)$ for some n.
- Related terms/ideas:
  - Equivalence classes.
  - Partition of a set.

Minute Essay

- If we define a relation ρ on the students in this class, such that $x \; \rho \; y$ iff x and y are sitting in the same row:

  Is ρ reflexive? symmetric? transitive? antisymmetric?

Slide 7