Slide 1

Administrivia

- Reminder: Homework 7 due Friday.

Slide 2

Minute Essay From Last Lecture

- Question: Given A and B as follows, compute $A \cdot B$:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 5 & 1 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- Answer?

Slide 3

Graphs — Overview

- In some contexts, "graph" means a plot of a function, other pictorial representation of data.
- In other contexts, it's an abstract idea meant to represent relationships among a set of things. Examples:
  - Hasse diagrams of chapter 4.
  - Airline route maps.
  - Simplified maps showing driving distances between cities.

  Common idea — set of things (set elements, cities) and a notion that some pairs of them are connected somehow. Details we don't care about have been "abstracted out".

Slide 4

Graphs — Definition

- Formal definition (undirected graph):
  - Nonempty set N of nodes (vertices).
  - Set A of arcs (edges).
  - Function $f : A \to \{\{x, y\} | x, y \in N\}$ (unordered pairs of nodes).

  Notice that we can have "loops" and also "parallel arcs".
- Variations/extensions:
  - Directed graph — edges are ordered pairs (i.e., "one-way").
  - Labeled graph — each vertex has some associated info ("label").
  - Weighted graph — each edge has some associated info ("weight").

Slide 5

Graphs — Terminology

- Adjacent nodes (arc from one to the other).
- Loop, parallel arc.
- Simple graph (no loops or parallel arcs).
- Complete graph (every pair of nodes adjacent).
- Path (sequence of arcs), connected graph.
- Cycle, acyclic graph.

Slide 6

Isomorphic Graphs

- What we care about is the relationship between nodes and arcs, not exact visual representation.
- Can formalize this as "isomorphism" — two graphs are isomorphic if one is just a "relabeling" of the other.
- Formal definition is in terms of one-to-one functions, one from nodes of graph G_1 to nodes of graph G_2 and one from arcs of graph G_1 to arcs of graph G_2. Idea is that if an arc connects two nodes in G_1, the corresponding arc in G_2 connects the corresponding nodes.

Slide 7

Computer-Friendly Representation of Graphs

- For humans, representing graphs pictorially usually works well. For computers, other representations work better.
- Key idea is to come up with a way to represent the essential information — set of nodes and which ones are connected.

Slide 8

Adjacency Matrices

- Idea is to put the n nodes in some (arbitrary) order and define an n-by-n matrix A such that A_{ij} is the number of arcs connecting node i and node j.
- For an undirected graph, what property does this matrix have? that it might or might not have for a directed graph?
- Variation: For a weighted graph with no parallel arcs, we could let A_{ij} be the weight of the arc from node i to node j.
- Example(s)?

Adjacency Lists

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- Idea is to again put n nodes in some arbitrary order, but rather than a matrix define an array of n lists, one for each node, with the list for node i containing all nodes j that are adjacent to node i. Parallel arcs mean "duplicate" entries.
- Example(s)?

Adjacency Matrix Versus Adjacency List

Slide 10

- Which uses less space?
- Which makes it faster to answer the question "is node i adjacent to node j?"

Slide 11

Minute Essay

- None — quiz.