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Administrivia

- Notice that my slides will be available linked from the “lecture topics and assignments” Web page, usually fairly soon after class.
- First homework on Web; due a week from Monday.

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Minute Essay From Last Lecture

- Question: What have you liked/disliked about previous math courses?
 - Answers indicate that people like/dislike different things! some “only if it applies to something I care about”, some “I like math”, some “I hate proofs”
...
- Notice that this is “discrete math” as opposed to the kind of “continuous math” involved in calculus.

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Why Study Propositional Logic?

- Because it's conceptually related to Boolean algebra (used in programming, circuit design, etc.).
- Because it's related to proofs, which you should know a bit about.
- As an example of a "formal system" — represent something symbolically, define and apply rules for manipulating symbols, etc. Other examples in automata theory, theory of databases, etc.
- Because when you ask Dr. Theory (Myers) what you should learn in this course, he says "logic, logic, logic, logic!"
- Because after logic, the rest of the course will (probably) seem easy!

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Propositional Logic — The Big Picture

- Underlying many fields is a notion of "valid argument", one thing "following logically" from another — math, science, law, etc. (Consider example at the start of chapter 1.)
- Can define precisely what this means using natural language, but it's difficult and clumsy.
- If we use mathematical notation instead, it's easier to produce/follow chains of reasoning.

(Analogous to "word problems" in algebra — the idea is to turn something that's clumsy to work with into mathematical symbols, operate on the symbols with well-defined math, and translate the result back into words.)

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Statements / Propositions

- Definition — something (in natural language, a sentence) that is either true or false. (We might not know which.)
- Which of these are statements?
 - Water is wet.
 - Water is not wet.
 - Is the sky blue on Venus?
 - There is life on Mars.
- Notational convention — A, B, C , etc., are statements.

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Connectives

- Can build up more complicated statements by combining simpler ones, using “connectives” — each has intuitive meaning, formal definition.
- “and” connective pretty clear — $A \wedge B$ defined by truth table
- “or” connective also pretty clear — $A \vee B$ defined by truth table
Notice that this is “inclusive or” — not always the same as what we mean by “or” in natural language.
- “not” connective also pretty clear — A' defined by truth table
- “implies” connective is trickier — $A \rightarrow B$ defined by truth table
Why define it that way? Stay tuned . . .
- “is equivalent to” connective also pretty clear — $A \leftrightarrow B$ defined by truth table

Why Did We Define Implication That Way?

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- Definition of $A \rightarrow B$ when A is true seems reasonable, right?
- When A is false, though — why say $A \rightarrow B$ is true?
 - “Benefit of the doubt” argument: We have to call it either true or false, and it’s not obviously false.
 - “It’s math” argument: Maybe this definition *doesn’t* express some fundamental truth. But in some sense math is its own universe, and we can define things any way we want (though we hope the definitions fit together in a nice way and maybe have applications).
(In fact, some treatments of propositional logic just define implication formally, in terms of other connectives, and don’t try to justify it.)

Compound Statements / Well-Formed Formulas

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- Natural-language equivalents of statements joined by connectives:
 - Water is wet and grass is green.
 - If Jo(e) is a CS major, Jo(e) must take this course.
- We can “nest” connectives, e.g., $(A \wedge B)'$.
- We can define a notion of “well-formed formula” (wff) based on this (formal definition should be recursive, and we’ll do that later) — basically, a “sensible” combination of statement letters, connectives, and parentheses.
- Notational convention — P, Q, \dots for wffs.
- We can use truth tables to figure out truth values for wffs. (How many rows do we need?) Let’s do an example . . .

Compound Statements, Continued

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- As an example, let's try turning the example at the start of chapter 1 into formulas, using the following:

A is "The client is guilty."

B is "The knife was in the drawer."

C is "Jason P. saw the knife."

D is "The knife was there on Oct. 10."

E is "The hammer was in the barn."

- And then we hope we can somehow use the formulas to help us decide whether the conclusion follows from the premises.

More Definitions

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- Some wffs are always true — "tautologies". Examples?
- Some wffs are always false — "contradictions". Examples?
- We can talk about two wffs P and Q being "equivalent" — $P \leftrightarrow Q$ is a tautology.

Write $P \leftrightarrow Q$.

Table of common equivalences on p. 8.

Additional widely-used equivalences — "De Morgan's Laws".

Propositional Formulas in Other Contexts

- Notice similarities between the connectives here and
 - Boolean expressions in programming languages.
 - Expressions for “advanced search” in some search engines, database queries, etc.
- Can use rules we have so far to simplify such expressions.

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Valid Arguments

- Now we want to capture notion of “valid argument” — formal version of what someone familiar with proofs would recognize as such.
- Idea is that we have “hypotheses” P_1, P_2, \dots, P_n and “conclusion” Q , and we want to know when we can be sure that the truth of the hypotheses guarantees the truth of the conclusion — i.e., when is

$$(P_1 \wedge \dots \wedge P_n) \rightarrow Q$$

a tautology?

- Could we use truth tables? If we can, would we always want to?

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Valid Arguments, Continued

- A more algorithmic view — apply “derivation rules” to construct a “proof sequence”.

Idea is that we have a list of wffs that we know are true any time all the hypotheses (P_1, P_2, \dots, P_n) are true. Then we proceed thus:

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1. Initialize this list to include just P_1, P_2, \dots, P_n .
2. If conclusion Q is on the list, stop.
3. Apply a derivation rule to one or more wffs in the list, producing a new wff X . Add X to the list.
4. Go to step ??.

Derivation Rules

- What kind of “derivation rules” would be good?
 - When we apply one, we want it to be the case that if the wffs we start with are true, the wff we derive is also true — system is “sound”. “Everything we can prove is true.”
 - Together they are powerful enough to allow us to construct proof sequences for all true statements — system is “complete”. “Everything that is true has a proof.” (Possible here, but not for more complicated kinds of logic!)

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Minute Essay

- Suppose we have
 W is "Water is wet."
 L is "There is life on Mars."
- Write as wffs the following:
"Water is wet and there is life on Mars."
"There is life on Mars if water is wet."

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Minute Essay Answer

- "Water is wet and there is life on Mars.":
 $W \wedge L$
- "There is life on Mars if water is wet.":
 $W \rightarrow L$

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