Slide 1

Administrivia

- Reminder: Homework 1 on Web (linked from “Lecture topics and assignments” page), due next Monday.

Slide 2

Propositional Logic, Review

- Define statements, connectives, wffs.
- Define “valid argument” / proof sequence:
  - Start with hypotheses.
  - Apply derivation rules until we get conclusion. Derivation rules can all be proved, and should seem plausible as well.

Slide 3

Derivation Rules

- Two groups of basic rules:
  - Equivalence rules (two-way) — p. 23.
  - Inference rules (one-way) — p. 24.

  ("Do I have to memorize these?" No. Exams and quizzes will be open book.)
- Formally, these rules are true because we can prove them using truth tables.
- They should also seem plausible, maybe even "obviously true".
- Can derive additional rules; table on p. 31 lists some.

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Building Blocks for Proof Sequences

- Equivalence rules (two-way), p. 23. Notice that these *can* be applied to parts of wffs.

  Example: "Implication" says that if we have $P \rightarrow Q$ we can replace it with $P' \vee Q$, or vice versa.
- Inference rules (one-way), p. 24. Notice that these *cannot* be applied to parts of wffs.

  Example: "Modus ponens" says if we have $P \rightarrow$ Q on one line, and P on another, we can write down a new line Q.
- "Deduction method": To show that $P_1, P_2, \ldots P_n$ guarantee conclusion $R \rightarrow Q$, we can show that $P_1, P_2, \ldots P_n, R$ guarantee Q
- Derived inference rules, p. 31. Notice that many of these are proved as problems, and you should only use them for later problems. (E.g., okay to use the results of problem 23 in problem 25, but not vice versa.)

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Proof Sequences — Simple Example

- So, let's do an example:
  - Hypotheses:

    A

    B
  - Conclusion:

    $(A \wedge B) \vee C$
- "Justifications" we write down for each step aren't technically required for a valid proof sequence. We put them in to help human readers.

Slide 6

Hints for Constructing Proof Sequences

- Two things involved in constructing proofs:
  - Applying the rules correctly — not so difficult, if you correctly match up your formula with the rule.
  - Knowing which rule to apply — more difficult, gets a little easier with practice. Also see hints ("heuristics") on p. 26:
    - Consider using modus ponens often.
    - Consider using De Morgan's laws to simplify (?) $(P \vee Q)'$, $(P \wedge Q)'$.
    - Consider using equivalence rules to convert $P \vee Q$ to $P' \rightarrow Q$.

    Sometimes helps to "work backward" — figure out an intermediate result from which you could reach the conclusion, then figure out how to get the intermediate result. If you do this, though, must still construct proof "going forward".

Examples

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- Lawyer example from start of chapter.
- Problem 25 in section 1.2.

So, What Does This Buy Us?

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- Yes, this can seem long and tedious. But . . .
- It's in some ways easier than other approaches, and certainly more reliable.
- Compare to "word problems" in algebra — first convert from natural language to math, apply math, convert back — with practice, easier and more reliable than guessing.
- In a way, we're replacing thinking with symbol manipulation!

Slide 9

Minute Essay

- How much of this (if any) looks familiar to you from other courses?