Slide 1

Administrivia

- Quiz 1 Wednesday (last 10 minutes of class). Open book / open notes. Will cover propositional logic only. Problem(s) similar to minute essays and homeworks. Worth 10 points (as compared to 100 for midterm).

Slide 2

Why Predicate Logic?

- Propositional logic captures some of what we need to talk about things logically, but not everything.
- Example from classical logic:

  "All humans are mortal. Socrates is human. Therefore Socrates is mortal."

  No way to express this in propositional logic.
- What we want to add is some way to express the idea of something being true "for all x" or "for at least one x".

Slide 3

Predicates

- Define "predicate" — boolean-valued function of one or more variables.
- Examples with integer variables:

  $P(x) = (x > 0)$

  $Q(x, y) = (x < y)$
- Examples with people variables:

  $P(x)$ means "x is a student in CSCI 1323".

  $Q(x, y)$ means "x is taller than y".

Slide 4

Quantifiers

- Universal quantification: $(\forall x)P(x)$ means "for all x, $P(x)$ is true."
- Existential quantification: $(\exists x)P(x)$ means "there exists an x such that $P(x)$ is true."
- How to decide whether such a statement is true? For propositional-logic connectives, we could write down a truth table for different values of the formulas being connected. That won't work here. (Why?)
- Instead, notion of a "domain of interpretation" — (non-empty) range of values for the variable, definition of predicate(s).

  $(\forall x)P(x)$ means — ?

  $(\exists x)P(x)$ means — ?

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A Few More Definitions

- Define "variables" (usually write them x, y, etc.) and "constants" (usually write them a, b, etc.) — elements of domain of interpretation.
- "Free variables" are those not within scope of a quantifier — e.g., x but not y in $(\forall y)P(x,y)$.
- Notice that we can change the variable in a quantification — it's a "dummy variable" – except we can't duplicate another variable.
- As in propositional logic, can define notion of well-formed formula (wff) — "sensible" combination of predicates, quantifiers, connectives from propositional logic, and parentheses.
- How to express "All men are mortal", etc?

Slide 6

Interpretations

- Expressions involving predicates are true/false depending on "interpretation" (analogous to assigning values to statements in propositional logic):
  - Domain of the interpretation (must not be empty).
  - Assignment of a property of objects in the domain to each predicate.
  - Assignment of a particular object to each constant symbol.
- Given an interpretation and an expression, we can (usually) compute a value for it. (What if there's at least one free variable?)

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Interpretations — Example

- Suppose the domain is the integers, and $Q(x)$ means "x has an integer square root".
- What is the "truth value" of the following?
  - $Q(4)$
  - $Q(2)$
  - $(\forall x)Q(x)$
  - $(\exists x)Q(x)$
  - $Q(4) \ \vee \ Q(2)$
  - $Q(c)$
  - $Q(x)$

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English to Formulas

- Given people as a domain and predicates
  - $C(x)$ meaning "x is a CS student"
  - $D(x)$ meaning "x must pass CSCI 1323 to graduate"
  - $B(x)$ meaning "x is a business major"
  - $M(x)$ meaning "x likes math"
- Translate:
  - "'All CS majors must pass CSCI 1323 to graduate."
  - "Some CS majors are business majors."
  - "Some CS majors like math."
  - "Not all CS majors like math."

Slide 9

Minute Essay

- Consider formulas $Q(a)$, $Q(b)$, $(\forall x)Q(x)$. Tell me whether each is true or false for the following interpretations.
- Interpretation 1: domain of interpretation is the integers, $a = 1$, $b = 2$, and $Q(x)$ means "$2x$ is an even integer".
- Interpretation 2: domain of interpretation is the rational numbers, $a = 1/2$, $b = 1$, and $Q(x)$ means "$2x$ is an even integer".

(Reminder: Homework 1 due by 5pm today. For those under new Honor Code — this is "pledged" work.)

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Minute Essay Answer

- Interpretation 1: domain of interpretation is the integers, $a = 1$, $b = 2$, and $Q(x)$ means "$2x$ is an even integer".
  $Q(a)$ true, $Q(b)$ true, $(\forall x)Q(x)$ true.
- Interpretation 2: domain of interpretation is the rational numbers, $a = 1/2$, $b = 1$, and $Q(x)$ means "$2x$ is an even integer".
  $Q(a)$ false, $Q(b)$ true, $(\forall x)Q(x)$ false.