Slide 1

Administrivia

- Homework 2 on Web. (PostScript and PDF versions are nicer-looking than HTML.)
- Quiz solutions available on the Web, usually shortly after class.

Slide 2

Universal Generalization (Corrected)

- Rule for introducing $\forall$. (Why do we want to do this?)
- If we have $P(x)$

  we can write $(\forall x)P(x)$

  provided x is "arbitrary" — not a free variable in a hypothesis, not a variable we got from ei, not a free variable in a formula we got from ei. (For last part, consider example at bottom of p. 49.)

  (Yes, this is tricky to understand/apply.)
- "If we know $P(x)$ for arbitrary x, then $P(x)$ for all x."
- Review problem 9 section 1.4.

Slide 3

Predicate Logic, Recap / What Next?

- Now we have a set of derivation rules for predicate logic (we'll add a few more for convenience later).
- As with propositional logic, we could show that these rules are "sound" (if we can prove something, it's true/valid) and "complete" (if something is true/valid, we can prove it).

Slide 4

Temporary Hypotheses

- In propositional logic, we allowed proving a conclusion of the form $P \rightarrow Q$ by adding P to the list of hypotheses and proving Q.
- Along the same lines, we allow "temporary hypotheses":
  Suppose as part of a proof we want to show that $R \rightarrow S$ follows from the hypotheses. If $R \rightarrow S$ is the conclusion, deduction method works. What if it's not? Then we can't just add R to the list of hypotheses. What to do?
- One solution would be (in mathspeak) a lemma ("branch" or side proof).

Slide 5

Temporary Hypotheses, Continued

- Another solution is basically an inline lemma:
  - Introduce “temporary hypothesis” T.
  - Derive some more steps from earlier results and T, ending with S.
  - Conclude that $T \rightarrow S$.

  Note that the formulas we derive from earlier steps and T might depend on T, so — indent to make it clear that they’re not part of the main proof.
- Example — section 1.4 problem 21.

Slide 6

One More Rule, a Conclusion

- One more rule — negation (example 32 p. 53).
- A conclusion — the goal of formal logic is to make arguments as meaningless as possible (!) — i.e., abstract out everything that doesn’t matter, and apply formal mathematical rules to what’s left.

Another Example

- Section 1.4 problem 31.
- (More examples next time.)

Slide 7

Minute Essay

- How was Homework 1? easy/hard, long/short, . . .

Slide 8