Administrivia

Slide 1

- None.

Proof Techniques — Overview/Rants

Slide 2

- By "proof" we mean informal version, sometimes relying on context, of formal "this follows from that" arguments of chapter 1.
- Goal is to convince human reader. Sometimes a sequence of formulas will do. Other times some prose is needed to explain what they mean. (Ask yourself: Would this make sense to you?)
- If you are asked to show, e.g., that if $x = 5$ then $x^2 = 25$, *please do not* start by writing $x^2 = 25$! (Don't write down as "true" things you haven't shown to be true!)

Slide 3

Exhaustive Proof / Proof By Cases

- Idea here is to prove by considering each "case" separately. Only works if there are finitely many. (Recall result from propositional logic that allows this.)
- Simple example: To show that for all integers x with $0 \leq x \leq 4$, $x^2 < 20$, five cases to consider.
- Slightly more complex example: To show something for all integers, can consider two cases, odd integers and even integers. (Aside: How shall we define "even"? Is zero even?)
- Much more complex example: Computer-assisted proof of 4-color map theorem (1976, used almost 2000 separate cases).

Slide 4

Direct Proof

- Idea here is to show $P \rightarrow Q$ like we've been doing — assume P and derive Q — but less formally.
- Example: Show that for integers p and m, if p is even and m is positive, p^m is even.

Proof by Contraposition

Slide 5

- Idea is based on a derived rule from propositional logic: If $Q' \rightarrow P'$, then $P \rightarrow Q$.

  So if proving $P \rightarrow Q$ is difficult, we can try proving $Q' \rightarrow P'$ instead.

- Example: Show that if m and n are integers and $m + n$ is even, either m and n are both even or m and n are both odd.

Proof By Contradiction

Slide 6

- Idea is based on another rule we could prove using propositional logic: If $(P \wedge Q') \rightarrow false$, then $P \rightarrow Q$.

  So if proving $P \rightarrow Q$ is difficult, we can try assuming $P \wedge Q'$ and "deriving a contradiction".

  Note that sometimes P is just $true$.

- Example: Show that $\sqrt{2}$ is irrational.

Slide 7

Minute Essay

- Find a counterexample for the following conjecture: “If x is an integer, $\sqrt{x}$ is an integer.”
- To show that there is no largest prime, we could assume P and derive a contradiction. What is P? (You don’t have to show there’s no largest prime, just say what P is.)
- (Reminder: Homework 2 due.)

Slide 8

Minute Essay Answer

- Find a counterexample for the following conjecture: “If x is an integer, $\sqrt{x}$ is an integer.”

  $x = 2$
- To show that there is no largest prime, we could assume P and derive a contradiction. What is P? (You don’t have to show there’s no largest prime, just say what P is.)

  “There is a largest prime.”