Slide 1

Administrivia

- Homework 3 on Web. Due next Wednesday.

Slide 2

Proof Techniques, Review/Recap

- To disprove "for all integers n, $P(n)$" just need one counterexample. To prove, must show true for all n.
- Techniques so far for proving $P \rightarrow Q$:
  - Exhaustive proof: Consider all possible cases where P is true.
  - Direct proof: Assume P and derive Q.
  - Proof by contraposition: Assume Q' and derive P'.
  - Proof by contradiction: Assume $P \wedge Q'$ and derive "contradiction" (something impossible).

Slide 3

First Principle of Mathematical Induction

- We can prove that $P(k)$ is true for all integers $k \geq N$ (often N is 0 or 1, but not always) if we can show:
  - Base case: $P(N)$
  - Inductive step: For $k \geq N$, $P(k) \rightarrow P(k+1)$
    That is: Assume $P(k)$ and $k \geq N$ ("inductive hypothesis"), and show that then $P(k+1)$
- For readability/clarity, make this explicit, especially what you assume / have to show for inductive step.
- Works because we have $P(N)$ and then a chain of implications: $P(N) \rightarrow P(N+1), P(N+1) \rightarrow P(N+2), \ldots$

Slide 4

First Principle of Mathematical Induction — Examples

- Example: Show that for $n \geq 1$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

- Example: Show that for $n \geq 1$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Slide 5

Second Principle of Mathematical Induction

- Can also show that $P(k)$ is true for all integers $k \geq N$ (often N is 0 or 1, but not always) if we can show that:
  - Base case: $P(N)$
  - Inductive step: For $k \geq N$, $((N \leq r \leq k) \rightarrow P(r)) \rightarrow P(k+1)$
    That is: Assume that $P(r)$ holds for all integers r with $N \leq r \leq k$, and that $k \geq N$ ("inductive hypothesis"), and show that then $P(k+1)$
- For readability/clarity, again make this explicit ...
- Notice — inductive hypothesis here is more complicated, but gives you more to work with.
- Works because we have $P(N)$ and then a chain of implications:
  $P(N) \rightarrow P(N+1), P(N) \wedge P(N+1) \rightarrow P(N+2), \ldots$

Slide 6

Second Principle of Mathematical Induction — Example

- Consider a perforated sheet of stamps. How many "tear into two sheets" operations are needed to produce single stamps?
- Conjecture, based on some examples — if there are n stamps ($n \geq 0$), we need $n-1$ operations.
- Can prove with second principle — to be continued.

Minute Essay

- None — quiz.