Slide 1

Administrivia

- Reminder: Quiz solutions on Web, usually shortly after class.
- Course "useful links" page has links to more examples of induction.

Slide 2

Mathematical Induction, Continued

- Basic idea is to prove something true for all integers greater than some base value (usually 0 or 1) in two steps:
  - Base case — prove directly for smallest value.
  - Inductive step — prove that if true for k (first principle), or all numbers from base case through k (second principle), then also true for $k + 1$.
- Works because the base case gives you a starting point, and the inductive step can be used to build up a sequence of implications, and then from propositional logic . . .
- Inductive step is conceptually similar to what you do in writing a recursive function/procedure — break up problem for $k + 1$ into "smaller problems" that you can "solve" with the inductive hypothesis.

Slide 3

Example From Last Time, Continued

- Consider a perforated sheet of stamps. How many “tear into two sheets” operations are needed to produce single stamps?
- Conjecture, based on some examples — if there are n stamps ($n \geq 0$), we need $n - 1$ operations.
- Can prove with second principle — to be continued.

Slide 4

More Examples

- Section 2.2 problem 31.

Minute Essay

- Prove using mathematical induction that for all $n \geq 1$

$$\sum_{i=1}^{n}(2i-1) = n^2$$

Slide 5

Minute Essay Answer

- Base case: $n = 1$. $n^2 = 1$ and

$$\sum_{i=1}^{n}(2i-1) = 1$$

- Inductive step: Assume

$$\sum_{i=1}^{k}(2i-1) = k^2$$

and show

$$\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$$

Using inductive hypothesis:

$$\sum_{i=1}^{k+1}(2i-1) = \sum_{i=1}^{k}(2i-1) + 2(k+1) - 1 = k^2 + 2k + 1 = (k+1)^2$$

Slide 6