Slide 1

Administrivia

- Reminder: Homework 4 due Monday.
- If you didn't do well on Quiz 3, remember that I drop your lowest quiz score.

Slide 2

Recursive Definitions — Review

- Recall examples from last time — recursive definitions of sequences, sets, operations, algorithms.
- For sets, notice that this means that to claim that something is in the set you need to be able to show that it's either a base case or can be obtained from a base case by applying one of the "rules" that define the set.

Slide 3

Recursive Algorithms, More Examples

- Two good examples in text — selection sort and binary search.
- Another example — “quicksort”.

```
// pre:  i, j are valid indices for L
// post:  L(i) through L(j) are "sorted"
qsort(list L, index i, index j)
        if (i >= j)
                return
        else
                elem pivot = L(i)
                // rearrange L(i+1) through L(j) s.t.:
                //      L(i) .. L(m-1) <= pivot
                //      L(m) = pivot
                //      L(m+1) .. L(j) >= pivot
                index m = split(pivot, L, i, j)
                qsort(L, i, m-1)
                qsort(L, m+1, j)
end qsort
```

(Why does this work?)

Slide 4

Recurrence Relations

- Recall the silly example of defining a sequence recursively:

$$S(1) = 1$$
$$S(n) = S(n-1) \times 10, \quad \text{for } n > 1$$

Expanding out some terms, it seems fairly obvious that an equivalent definition would be $S(n) = 10^{n-1}$ — a “closed-form solution” to the recurrence relation given in the second line of the definition.

- We’ll look at various ways to get from a recursive definition to a closed-form one, because the latter are easier to compute, but sometimes it will be much easier to write down the definition recursively.

Slide 5

Solving Recurrence Relations, Continued

- For the silly example

$$\begin{aligned} S(1) &= 1 \\ S(n) &= S(n-1) \times 10, \quad \text{for } n > 1 \end{aligned}$$

  we guessed a solution of $S(n) = 10^{n-1}$. Can we verify that this is the same as the recursive definition? yes, via a proof by induction . . .

- Try another example — section 2.4 problem 75.
- Call this method “expand, guess, verify”.

Slide 6

Solving Recurrence Relations, Continued

- Is there another way? In general, probably not, but there are some formulas for some frequently-occurring special cases.
- One is “first-order linear” recurrence relations. If

$$S(n) = cS(n-1) + g(n)$$

  then we can show (see textbook for derivation) that

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n}(c^{n-i}g(i))$$

- Apply this to the two problems we did earlier — we should get the same results.

Slide 7

Minute Essay

- Consider the following recursive definition of a sequence:

$$\begin{aligned} S(1) &= 1 \\ S(n) &= 10S(n-1)+1, \quad \text{for } n > 1 \end{aligned}$$

What are $S(1), S(2), \ldots S(5)$?

Slide 8

Minute Essay Answer

- The first few terms:

$S(1) = 1$

$S(2) = 11$

$S(3) = 111$

$S(4) = 1111$

$S(5) = 11111$