Slide 1

Administrivia

- Quiz 4 moved to Friday.

Slide 2

Sets — Review

- Last Monday we reviewed/defined a lot of stuff about sets:
  - Ways to write/specify them.
  - Subsets and power sets.
  - Operations on sets.
  - Countability.
- Judging by minute essay, most stuff is familiar, with the exception of set difference ($A - B$).

Slide 3

Countable and Uncountable Sets, Just a Bit More

- We said last week that we'd say two sets are "the same size" if we could set up a one-to-one correspondence between them.
- We also said that we'd say a set S is countable if there's some way to write down all elements "in order" — i.e., set up a one-to-one correspondence with a subset of the positive integers. Surprisingly many sets are countable — e.g., the rational numbers.
- Not all sets are countable, though — e.g., $\mathbb{R}$ is not. Proof is by contradiction: Suppose it is, and then find a real number that isn't counted. (Details in textbook.)
- We can also prove that S and $\mathscr{P}(S)$ are not "the same size", again by contradiction ("Cantor's theorem").

Slide 4

Counting (Combinatorics)

- "Counting" sounds too trivial for a college-level course, right? but consider situations in which you want to know how many things there are in a set but don't actually want to list them all:
  - Given what a password is supposed to look like (4 digits, 20 characters, etc.), how many are there? i.e., how easy would it be to guess?
  - Given a scheme for IP addresses, how many are possible? i.e., are there enough for everything we want to give one to?

Slide 5

Multiplication Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2, how many outcomes are there for the sequence "event 1, then event 2"?
- Pictorially, we could draw a tree, and then we can see there are $N_1 \times N_2$.
- This is easily extended by induction to sequences of more than two events.
- Example: If a password consists of 4 decimal digits, how many are there? (And if we allowed 10 seconds to try each one, how long would it take to try them all?)
- Example: If a license-plate number is 3 decimal digits followed by three alphabetic characters, how many are there?

Slide 6

Addition Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2 (and the sets of "event 1 outcomes" and "event 2 outcomes" are disjoint), how many outcomes are there for the event "event 1 or event 2"?
- Fairly easy to see that there are $N_1 + N_2$ possibilities in all.
- This also is easily extended by induction to combinations of more than two events.
- Example: If you have to choose an elective from either the Department of Esoteric Pursuits (which offers 10 of them) or from the Department of Life Skills (which offers 6 of them), how many choices are there in all? (assuming no courses are cross-listed).

Combining the Addition and Multiplication Principles

- Example: How many phone numbers are there that have either area code 210 or area code 512?
- Example: How many 7-digit phone numbers have at least one repeated digit?

Slide 7

Decision Trees

- Sometimes it's useful to represent a sequence of choices as a "decision tree" and explicitly count leaf nodes.
- Example: How many ways are there to get 4 coin tosses with no sequences of three heads or three tails?

Slide 8

More Examples

- Section 3.2 problem 40.
- Section 3.2 problem 44.
- Section 3.2 problem 58.

Slide 9

Minute Essay

- If a password is at least 5 characters and no more than 8, where a character is either a digit or a lowercase character, how many possible passwords are there? (Okay to just write down an expression and not simplify — e.g., 10^4.)

Slide 10

Minute Essay Answer

- There are $10 + 26 = 36$ choices for each character in a password. Thus, there are 36^n choices for an n-character password. If we allow lengths from 5 to 8 inclusive, that gives

$$36^5 + 36^6 + 36^7 + 36^8$$

possibilities in all.