Slide 1

Administrivia

- Homework 6 (on chapter 3) on Web. Due next Friday.
- Reminder: Quiz 4 Friday.

Slide 2

Counting, Recap/Review

- Multiplication principle — if there are N ways to do one thing, and M ways to do another, there are $N \times M$ ways to do first one and then the other.
- Addition principle — if there are N ways to do one thing, and M ways to do another, there are $N + M$ ways to do one or another.
- Can combine these in interesting and effective ways. Recall examples from last time.
- Decision trees also sometimes useful. Recall example from earlier class (sequences of heads and tails).

Slide 3

Principle of Inclusion/Exclusion

- Motivating(?) example:

  You take a poll of how many people support propositions A and B. You find that 10 of them support A, 20 support B, and 5 support both A and B. How many support either A or B?

- Using set notation, with $|S|$ meaning the number of elements in S:

  Given $|A| = 10$, $|B| = 20$, and $|A \cap B| = 5$,

  what is $|A \cup B|$?

- We can use the addition principle to derive

$$|A \cup B| = |A| + |B| - |A \cap B|$$

  (Intuitive idea is that we count everything in both sets, and in doing that we count some things twice, so we must correct.)

Slide 4

Principle of Inclusion/Exclusion, Continued

- What if there were three propositions/sets? Can we derive a rule?
- Sure . . . (next slide).

Slide 5

Principle of Inclusion/Exclusion, Continued

- Rule for three sets is

  $$|A \cup B \cup C| = |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$$

- Intuitive idea:

  Count all the A's, all the B's, all the C's.

  A&B's, B&C's, and A&C's have been counted twice; A&B&C's have been counted three times.

  Subtract counts of A&B's, B&C's, and A&C's; now A&B&C's have been counted zero times.

  Add count of A&B&C's.

- Formally, derive from rule for two sets and rules for set operations.

Slide 6

Principle of Inclusion/Exclusion, Continued

- There's a pattern, captured in general form of rule (p. 205). (In another textbook — "A Ghastly Formula".)
- For more interesting examples (most beyond the scope of this course, Google "inclusion/exclusion principle").

Slide 7

Pigeonhole Principle

- Idea is that if you have n items placed in k bins, and $n > k$, then at least one bin has more than one item.

  Converse is that if no bin contains more than one item, n can be at most — what?

  More general version — if you have k bins and more than mk items, there's at least one bin with more than m items.

- Example — section 3.3 problem 17.

Slide 8

Pigeonhole Principle, Continued

- Another example (discovered on a Web page at Stanford):

  If A is a set of 10 integers in the range 1 to 100, show that there are at least two distinct and disjoint subsets of A that have the same sum.

  (Idea is to count number of possible subsets and also figure out range of potential sums. If more subsets than possible sums . . .)

Minute Essay

- If you have six integers in the range from 1 to 10 inclusive, can you be sure that at least two of them have an odd sum? (E.g., it's true for the integers 1 through 6, since 1 plus 2 is odd.)

Slide 9

Minute Essay Answer

- Yes — there are 5 even numbers in the range 1 through 10 and 5 odd numbers, so if you pick 6 numbers you'll have at least one of each, guaranteeing a pair with an odd sum.

Slide 10