Slide 1

Administrivia

- Quiz 5 moved to next Monday.
- Homework 6 due date moved to this Friday.

Slide 2

Permutations and Combinations — Eliminating Duplicates

- In general it can be interesting to try to figure out how to “eliminate duplicates” — i.e., account for the fact that one way of counting things produces a lot of duplicate results.
- Example: How many ways can we rearrange the letters in the word “voodoo”?

Slide 3

Permutations and Combinations With Repetitions

- Definitions of $P(n, r)$ and $C(n, r)$ specified “without repeats”. What if we want to allow repeats?
- Permutations: How many ways can we choose an ordered sequence of r things from n possibilities, if we allow repeats? (Not too tough, right?)
- Combinations: How many ways can we choose an unordered collection of r things from n possibilities, if we allow repeats? This is trickier. We’ll use a clever idea from example 58.

Slide 4

Permutations and Combinations, More Examples

- Section 3.2, problem 25.
- Section 3.4 problem 31.

Slide 5

Probability — Equally-Likely Outcomes

- Basic definition: If S ("sample space") is a set of equally likely outcomes of some action (e.g., possible results of tossing a fair coin), and E ("event") is a subset of S, then we define the probability of E as

$$P(E) = \frac{|E|}{|S|}$$

  Examples: Sequences of coin tosses, 5-card "hands" chosen from 52-card deck, etc.

- Note that $0 \leq P(E) \leq 1$. (Why?) When is $P(E) = 0$? When is $P(E) = 1$?
- Note that we can apply anything we know about sizes of sets. (E.g., if E_1 and E_2 are disjoint, what is $P(E_1 \cup E_2)$ in terms of $P(E_1)$ and $P(E_2)$?)

Slide 6

Minute Essay

- Given 20 words, how many 6-word phrases can you make up, if no repeated words are allowed? ("refrigerator magnet poetry")

  Okay to express answers in terms of $P(n, r)$ and/or $C(n, r)$ or factorials.
- Suppose you select 6 marbles at random from a jar containing red, blue, yellow, and green marbles (at least 6 each). How many ways can this selection be made?

Minute Essay Answer

- Order matters here, so $P(20, 6)$
- Order doesn't matter here, but repetitions are allowed, so this is a case of "combinations with repetitions", so there are $C(6 + 4 - 1, 6)$ (=$C(9, 6)$) ways to select.

Slide 7