Slide 1

Administrivia

- Reminder: Quiz 5 Monday.
- Homework 7 on Web. Due next Friday. One more short homework.

Slide 2

Average-Case Analysis of Algorithms

- Previously we talked about estimating worst-case execution time of algorithms — amount of "work" as a function of input size.
- We could also talk about average-case amount of work, based on idea of expected value: Sample space is set of all possible inputs. For input x, $X(x)$ is the amount of work for x and $p(x)$ is the probability of x.

  Example — example 68 in textbook.

Slide 3

Binary (and other) Relations

- Idea of a binary relation is to express relationship between pairs of elements of a set. Some interesting special cases:
  - Partial orderings — useful in working out how we could put things "in order", e.g., a set of tasks with (some) ordering dependencies.
  - Functions (of 1 variable).
- Generalization — "n-ary relation", also with interesting special cases:
  - Functions of more than 1 variable.
  - Relational databases.

Slide 4

Binary Relations

- Formal definition: A binary relation ρ on a set S is a subset of $S \times S$. Usually this set is defined by some property of interest. For $a, b \in S$, we write $a \ \rho \ b$ iff (if and only if) (a, b) is in this subset.
- Examples:
  - S is people in the world; $x \ \rho \ y$ iff x and y are siblings.
  - S is integers; $x \ \rho \ y$ iff $x < y$.
  - S is integers; $x \ \rho \ y$ iff y is a multiple of x.
  - S is sets of integers; $x \ \rho \ y$ iff $x \subseteq y$.
- Notice that for a given relation ρ and element x, there can be any number (including zero) of y's such that $x \ \rho \ y$ and any number (including zero) of y's such that $y \ \rho \ x$.

Slide 5

Properties of Binary Relations

- ρ is *reflexive* if $x \ \rho \ x$ for all $x \in S$.
- ρ is *symmetric* if $(x \ \rho \ y) \ \rightarrow \ (y \ \rho \ x)$ for all $x, y \in S$.
- ρ is *transitive* if $(x \ \rho \ y) \ \wedge \ (y \ \rho \ z) \ \rightarrow \ (x \ \rho \ z)$ for all $x, y, z \in S$.
- ρ is *antisymmetric* if $(x \ \rho \ y) \ \wedge \ (y \ \rho \ x) \ \rightarrow \ (x = y)$ for all $x, y \in S$.
- Can combine these in interesting ways . . .

Slide 6

Partial Ordering

- Idea: Generalize idea of "ordering" to include relations where not all pairs of elements can be ordered.
- Definition: ρ is a partial ordering if it's reflexive, antisymmetric, and transitive.
- Examples: $\leq$ on integers or reals, $\subseteq$ on sets.

Slide 7

Equivalence Relation

- Idea: Generalize idea of "equals" to include relations where pairs of elements are equivalent but not identical.
- Definition: ρ is an equivalence relation if it's reflexive, symmetric, and transitive.
- Examples: $=$ on integers or reals, $(x \bmod n) = (y \bmod n)$ for some n.

Slide 8

Minute Essay

- If we define a relation ρ on the students in this class, such that $x \ \rho \ y$ iff x and y are sitting in the same row:

  Is ρ reflexive? symmetric? transitive? antisymmetric?
- Reminder — Homework 6 due today at 5pm.

Slide 9

Minute Essay Answer

- ρ is reflexive, symmetric, and transitive, but not antisymmetric.