Slide 1

Administrivia

- (None?)

Slide 2

Binary Relations — Review

- Idea of a binary relation is to express relationship between pairs of elements of a set. Formal definition is in terms of sets of ordered pairs.
- Several properties of interest:
  - ρ is *reflexive* if $x \ \rho \ x$ for all $x \in S$.
  - ρ is *symmetric* if $(x \ \rho \ y) \ \rightarrow \ (y \ \rho \ x)$ for all $x, y \in S$.
  - ρ is *transitive* if $(x \ \rho \ y) \ \wedge \ (y \ \rho \ z) \ \rightarrow \ (x \ \rho \ z)$ for all $x, y, z \in S$.
  - ρ is *antisymmetric* if $(x \ \rho \ y) \ \wedge \ (y \ \rho \ x) \ \rightarrow \ (x = y)$ for all $x, y \in S$.

Slide 3

Partial Ordering

- Idea: Generalize idea of “ordering” to include relations where not all pairs of elements can be ordered.
- Definition: ρ is a partial ordering if it’s reflexive, antisymmetric, and transitive.
- Examples: $\leq$ on integers or reals, $\subseteq$ on sets.
- If finite, can represent with “Hasse diagram” (see examples in textbook).
- Related terms:
  - Successor, predecessor, immediate predecessor, immediate successor.
  - Least, greatest elements.
  - Minimal, maximal elements.

Slide 4

Equivalence Relation

- Idea: Generalize idea of “equals” to include relations where pairs of elements are equivalent but not identical.
- Definition: ρ is an equivalence relation if it’s reflexive, symmetric, and transitive.
- Examples: $=$ on integers or reals, $(x \bmod n) = (y \bmod n)$ for some n.
- Related terms/ideas:
  - Equivalence classes.
  - Partition of a set.

Slide 5

Closures

- We can also talk about the “closure” of a relation with respect to a property — the smallest superset of the relation that has the property.
- Example: Define relation ρ on integers such that $x \ \rho \ y$ iff $y = x + 1$. What is the transitive closure of ρ?

Slide 6

Uses of Partial Orderings

- One thing a partial ordering (reflexive, symmetric, transitive relation — think “generalized $\leq$”) can express — ordering constraints among tasks.
- We’ll look at one application — topological sorting. PERT charts discussed in book.

Slide 7

Topological Sorting

- Idea here is to take a partial ordering and find a way to extend it to a "total" ordering (i.e., add pairs so that for every x and y either $x\ \rho\ y$ or $y\ \rho\ x$. How is this useful? e.g., find a way to "schedule" interdependent tasks.
- Notice that there could be more than one way to do this for a given partial ordering.
- How to do this? Next slide . . . (May not be covered in class.)

Slide 8

Topological Sorting, Continued

- Algorithm for finding *a* way to extend a partial ordering — "topological sort":
- Start with set S and partial ordering ρ on S. Idea is to turn S into a sequence $x_1, x_2, \ldots$ such that $(x_i\ \rho\ x_j) \ \rightarrow\ (i \leq j)$.
- The algorithm might look like this in pseudocode:

  while (S not empty)
  pick a minimal element x in S
  make it the next element of the sequence and remove it from S
  end while

- Does this work? i.e., does it produce an ordering that extends ρ? True if we can be sure that for x and y with $x\ \rho\ y$ x is picked before y.

Minute Essay

- None — quiz.

Slide 9