Slide 1

Administrivia

- Notice that my slides will be available linked from the “lecture topics and assignments” Web page, usually fairly soon after class. Usually there will be at least a preliminary version available before class as well.
- It will probably help to bring the textbook to class most days.

Slide 2

Minute Essay From Last Lecture

- Question: What have you liked/disliked about previous math courses?

  Answers indicate that people like/dislike different things! some people really liked calculus (or trig, or geometry), some hated it; some like math, some only if it has applications; some like proofs, some hate them.

  Some mentioned wanting to learn what “discrete structures” are. Maybe a better name for the course would be “discrete math” or “discrete math for computing”.

  Some mentioned graphs. Notice that “graphs” in this course are not “graphs of functions”.
- Programming background not an explicit prerequisite, but will help with some topics.

Slide 3

Why Study Propositional Logic?

- Because it's conceptually related to Boolean algebra (used in programming, circuit design, etc.).
- Because it's related to proofs, which you should know a bit about.
- As an example of a "formal system" — represent something symbolically, define and apply rules for manipulating symbols, etc. Other examples in automata theory, theory of databases, etc.
- Because when you ask Dr. Theory (Myers) what you should learn in this course, he says "logic, logic, logic, logic!"
- Because after logic, the rest of the course will (probably) seem easy!

Slide 4

Propositional Logic — The Big Picture

- Underlying many fields is a notion of "valid argument", one thing "following logically" from another — math, science, law, etc. (Consider example at the start of chapter 1.)
- Can define precisely what this means using natural language, but it's difficult and clumsy.
- If we use mathematical notation instead, it's easier to produce/follow chains of reasoning.

  (Analogous to "word problems" in algebra — the idea is to turn something that's clumsy to work with into mathematical symbols, operate on the symbols with well-defined math, and translate the result back into words.)
- Emphasis in this course is on the logic/math part rather than on translating real-world English into symbols.

Slide 5

Statements / Propositions

- Definition — something (in natural language, a sentence) that is either true or false. (We might not know which.)
- Which of these are statements?
  - Water is wet.
  - Water is not wet.
  - Is the sky blue on Venus?
  - There is life on Mars.
- Notational convention — A, B, C, etc., are statements.

Slide 6

Connectives

- Can build up more complicated statements by combining simpler ones, using "connectives" — each has intuitive meaning, formal definition.
- "and" connective pretty clear — $A \wedge B$ defined by truth table
- "or" connective also pretty clear — $A \vee B$ defined by truth table

  Notice that this is "inclusive or" — not always the same as what we mean by "or" in natural language.
- "not" connective also pretty clear — A' defined by truth table
- "implies" connective is trickier — $A \rightarrow B$ defined by truth table

  Why define it that way? Stay tuned ...
- "is equivalent to" connective also pretty clear — $A \leftrightarrow B$ defined by truth table

Slide 7

Why Did We Define Implication That Way?

- Definition of $A \rightarrow B$ when A is true seems reasonable, right?
- When A is false, though — why say $A \rightarrow B$ is true?
  - "Benefit of the doubt" argument: We have to call it either true or false, and it's not obviously false.
  - "It's math" argument: Maybe this definition *doesn't* express some fundamental truth. But in some sense math is its own universe, and we can define things any way we want (though we hope the definitions fit together in a nice way and maybe have applications).
    (In fact, some treatments of propositional logic just define implication formally, in terms of other connectives, and don't try to justify it.)

Slide 8

Compound Statements / Well-Formed Formulas

- Natural-language equivalents of statements joined by connectives:
  - Water is wet and grass is green.
  - If Jo(e) is a CS major, Jo(e) must take this course.
- We can "nest" connectives, e.g., $(A \wedge B)'$.
- We can define a notion of "well-formed formula" (wff) based on this (formal definition should be recursive, and we'll do that later) — basically, a "sensible" combination of statement letters, connectives, and parentheses.
- Notational convention — P, Q, … for wffs.
- We can use truth tables to figure out truth values for wffs. (How many rows do we need?) Let's do an example …

Slide 9

Compound Statements, Continued

- As an example, let's try turning the example at the start of chapter 1 into formulas, using the following:

  A is "The client is guilty."

  B is "The knife was in the drawer."

  C is "Jason P. saw the knife."

  D is "The knife was there on Oct. 10."

  E is "The hammer was in the barn."

- And then we hope we can somehow use the formulas to help us decide whether the conclusion follows from the premises.

Slide 10

More Definitions

- Some wffs are always true — "tautologies". Examples?
- Some wffs are always false — "contradictions". Examples?
- We can talk about two wffs P and Q being "equivalent" — $P \leftrightarrow Q$ is a tautology.

  Write $P \Leftrightarrow Q$.

  Table of common equivalences on p. 8.

  Additional widely-used equivalences — "De Morgan's Laws" (p. 9).

Slide 11

Propositional Formulas in Other Contexts

- Notice similarities between the connectives here and
  - Boolean expressions in programming languages.
  - Expressions for "advanced search" in some search engines, database queries, etc.
- Can use rules we have so far to simplify such expressions.

Slide 12

Minute Essay

- Part 1:
  - Why are you taking this course? CS major, CS minor, common curriculum?
  - Do you have any programming background?
- Part 2:

  Suppose we have
  - W is "Water is wet."
  - L is "There is life on Mars."

  Write as wffs the following:
  - "Water is wet and there is life on Mars."
  - "There is life on Mars if water is wet."

Slide 13

Minute Essay Answer

(For part 2):

- "Water is wet and there is life on Mars.":

  $W \;\wedge\; L$

- "There is life on Mars if water is wet.":

  $W \;\rightarrow\; L$