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### Administrivia

- Reminder: Homework 2 due Monday.
- One-page summaries of rules for propositional and predicate logic on Web (“Useful links and other resources” page). Also distributed in class.
- Quiz solutions on Web. Homework solutions distributed on paper only.
- Note that we will skip 1.6 for now, and 1.5 completely.

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### Homework 1 Review

- Showing that something is a tautology using a truth table:  
Goal is to evaluate formula for all ways of assigning true/false to statements  
— so how many lines?  
What should you conclude if you get “false” for some line?
- In proof sequences, be sure to explicitly number lines (for readability).  
Applying rules should be strictly mechanical (substitute expressions you have for  $P$ ,  $Q$ , etc., in rule). Remember that only equivalence rules apply to sub-formulas.  
Strictly speaking, apply only one rule per step. I’ll allow omitting steps, but only when it’s very clear; avoid if not sure.

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### Predicate Logic, Recap / What Next?

- Now we have a set of derivation rules for predicate logic (we'll add a few more for convenience later).
- As with propositional logic, we could show that these rules are “sound” (if we can prove something, it's true/valid) and “complete” (if something is true/valid, we can prove it).

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### Temporary Hypotheses

- In propositional logic, we allowed proving a conclusion of the form  $P \rightarrow Q$  by adding  $P$  to the list of hypotheses and proving  $Q$ .
- Along the same lines, we allow “temporary hypotheses”:  
Suppose as part of a proof we want to show that  $R \rightarrow S$  follows from the hypotheses. If  $R \rightarrow S$  is the conclusion, deduction method works. What if it's not? Then we can't just add  $R$  to the list of hypotheses. What to do?
- One solution would be (in mathspeak) a lemma (“branch” or side proof).

### Temporary Hypotheses, Continued

- Another solution is basically an inline lemma:
  - Introduce “temporary hypothesis”  $T$ .
  - Derive some more steps from earlier results and  $T$ , ending with  $S$ .
  - Conclude that  $T \rightarrow S$ .

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Note that the formulas we derive from earlier steps and  $T$  might depend on  $T$ , so — indent to make it clear that they’re not part of the main proof.

- Example — section 1.4 problem 21.

### One More Rule, a Conclusion

- One more rule — negation (example 32 p. 53).
- A conclusion — the goal of formal logic is to make arguments as meaningless as possible (!) — i.e., abstract out everything that doesn’t matter, and apply formal mathematical rules to what’s left.

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### Another Example

- Section 1.4 problem 31.
- (To be continued, and more examples next time.)

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### Minute Essay

- Are you keeping up with the reading, and is it helpful?
- Are you finding it helpful to do the “not to turn in” problems?

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