Slide 1

Administrivia

- Homework 3 on Web; due next Friday.

Slide 2

Minute Essay From Last Lecture

- Several wrap-up questions about propositional and predicate logic. To review briefly:
- In translating English to formulas — when do you use $\wedge$ and when $\longrightarrow$?
- Are there formal steps for finding a counterexample? not really, but there are some things that can help
- Exactly when can predicate logic rules (ei, ui, eg, ug) be used?
- More practice with predicate logic? try practice problems, and/or come talk to me.

Slide 3

Proof Techniques — Overview Again

- Up to now, we've done proofs in a very formal and structured way, and proved abstract ("meaningless") formulas.
- Now we turn to proofs done with less detail, less structure, more like those found in other fields (math, CS, science).
- Underlying ideas the same — demonstrate that something is true via a logical argument, reasoning from hypotheses to a conclusion. Difference is that we won't be as formal — so proofs will be shorter, but less like a game with well-defined rules.

Slide 4

Proof Techniques — Review

- To prove a conjecture false, only need one counter-example. To prove true, must prove for all possible cases.
- Recall from last time two techniques — exhaustive proof ("by cases"), direct proof.

Slide 5

Proof by Contraposition (Review)

- Idea is based on a derived rule from propositional logic: If $Q' \rightarrow P'$, then $P \rightarrow Q$.

  So if proving $P \rightarrow Q$ is difficult, we can try proving $Q' \rightarrow P'$ instead.

- Notice — contrapositive ($Q' \rightarrow P'$) is not the same as converse ($Q \rightarrow P$). No guarantees about whether, if $Q \rightarrow P$ we also know $P \rightarrow Q$.

- Example: Show that if m and n are integers and $m + n$ is even, either m and n are both even or m and n are both odd.

Slide 6

Proof By Contradiction

- Idea is based on another rule we could prove using propositional logic: If $(P \wedge Q') \rightarrow false$, then $P \rightarrow Q$.

  So if proving $P \rightarrow Q$ is difficult, we can try assuming $P \wedge Q'$ and "deriving a contradiction".

  Note that sometimes P is just $true$.

- Example: Show that $\sqrt{2}$ is irrational.

Minute Essay

- Find a counterexample for the following conjecture: "If x is an integer, $\sqrt{x}$ is an integer."
- To show that there is no largest prime, we could assume P and derive a contradiction. What is P? (You don't have to show there's no largest prime, just say what P is.)

Slide 7

Minute Essay Answer

- Find a counterexample for the following conjecture: "If x is an integer, $\sqrt{x}$ is an integer."

  $x = 2$
- To show that there is no largest prime, we could assume P and derive a contradiction. What is P? (You don't have to show there's no largest prime, just say what P is.)

  "There is a largest prime."

Slide 8