Slide 1

Administrivia

- Homework 7 on Web; due next Wednesday.

Slide 2

A Bit More About Cardinality of Sets

- Last time we defined a notion of “size” for infinite sets, and argued that:
  - $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ are all the “same size” (same cardinality — countably infinite).
  - $\mathbb{R}$, however, is larger (uncountable).
- We can also prove that S and $\mathscr{P}(S)$ are not “the same size”, again by contradiction. (“Cantor’s theorem”)

Slide 3

Counting (Combinatorics)

- "Counting" sounds too trivial for a college-level course, right? but consider situations in which you want to know how many things there are in a set but don't actually want to list them all:
  - Given what a password is supposed to look like (4 digits, 20 characters, etc.), how many are there? i.e., how easy would it be to guess?
  - Given a scheme for IP addresses, how many are possible? i.e., are there enough for everything we want to give one to?

Slide 4

Multiplication Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2, how many outcomes are there for the sequence "event 1, then event 2"?
- Pictorially — draw a tree. Clear that there are $N_1 \times N_2$.
- Can easily extend by induction to sequences of more than two events.
- Example: If a password consists of 4 decimal digits, how many are there? (And if we allowed 10 seconds to try each one, how long would it take to try them all?)
- Example: If a license-plate number is 3 decimal digits followed by three alphabetic characters, how many are there?

Slide 5

Addition Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2 (and the sets of "event 1 outcomes" and "event 2 outcomes" are disjoint), how many outcomes are there for the event "event 1 or event 2"?
- Fairly easy to see that there are $N_1 + N_2$ possibilities in all.
- Can also easily extend by induction to combinations of more than two events.
- Example: If you have to choose an elective from either the Department of Esoteric Pursuits (which offers 10 of them) or from the Department of Life Skills (which offers 6 of them), how many choices are there in all (assuming no courses are cross-listed)?

Slide 6

Combining the Addition and Multiplication Principles

- Example: How many phone numbers are there that have either area code 210 or area code 512?
- Example: How many 7-digit phone numbers have at least one repeated digit?

Decision Trees

- Sometimes it's useful to represent a sequence of choices as a "decision tree" and explicitly count leaf nodes.
- Example: How many ways are there to get 4 coin tosses with no sequences of three heads or three tails?

Slide 7

More Examples

- Section 3.2 problems 40, 44, 58.

Slide 8

Minute Essay

- If a password is at least 5 characters and no more than 8, where a character is either a digit or a lowercase character, how many possible passwords are there? (Okay to just write down an expression and not simplify — e.g., 10^4.)

Slide 9

Minute Essay Answer

- There are $10 + 26 = 36$ choices for each character in a password. Thus, there are 36^n choices for an n-character password. If we allow lengths from 5 to 8 inclusive, that gives

$$36^5 + 36^6 + 36^7 + 36^8$$

possibilities in all.

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