Administrivia

- Reminder: Homework 8 due today at 5pm.

- In the reading for today (4.1), it's okay to skim/skip the material on closures of relations (pp. 252–253) and Hasse diagrams (pp. 255–256).

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Binary (and other) Relations

- Idea of a binary relation is to express relationship between pairs of elements of a set. Some interesting special cases:
  - Partial orderings — useful in working out how we could put things "in order", e.g., a set of tasks with (some) ordering dependencies.
  - Functions (of 1 variable).
- Generalization — "n-ary relation", also with interesting special cases:
  - Functions of more than 1 variable.
  - Relational databases.

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Binary Relations

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- Formal definition: A binary relation ρ on a set S is a subset of $S \times S$. Usually this set is defined by some property of interest. For $a, b \in S$, we write $a \ \rho \ b$ iff (if and only if) (a, b) is in this subset.

- Examples:
    - S is people in the world; $x \ \rho \ y$ iff x and y are siblings.
    - S is integers; $x \ \rho \ y$ iff $x < y$.
    - S is integers; $x \ \rho \ y$ iff y is a multiple of x.
    - S is sets of integers; $x \ \rho \ y$ iff $x \subseteq y$.

- Notice that for a given relation ρ and element x, there can be any number (including zero) of y's such that $x \ \rho \ y$ and any number (including zero) of y's such that $y \ \rho \ x$.

Properties of Binary Relations

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- ρ is *reflexive* if $x \ \rho \ x$ for all $x \in S$.

- ρ is *symmetric* if $(x \ \rho \ y) \ \rightarrow \ (y \ \rho \ x)$ for all $x, y \in S$.

- ρ is *transitive* if $(x \ \rho \ y) \ \wedge \ (y \ \rho \ z) \ \rightarrow \ (x \ \rho \ z)$ for all $x, y, z \in S$.

- ρ is *antisymmetric* if $(x \ \rho \ y) \ \wedge \ (y \ \rho \ x) \ \rightarrow \ (x = y)$ for all $x, y \in S$.

- Can combine these in interesting ways . . .

Partial Ordering

- Idea: Generalize idea of "ordering" to include relations where not all pairs of elements can be ordered.

- Definition: ρ is a partial ordering if it's reflexive, antisymmetric, and transitive.

- Examples: $\leq$ on integers or reals, $\subseteq$ on sets.

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Equivalence Relation

- Idea: Generalize idea of "equals" to include relations where pairs of elements are equivalent but not identical.

- Definition: ρ is an equivalence relation if it's reflexive, symmetric, and transitive.

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- Examples: $=$ on integers or reals, $(x \bmod n) = (y \bmod n)$ for some n.

- Related terms/ideas:

  - Equivalence classes.

  - Partition of a set.

Uses of Partial Orderings

- One thing a partial ordering (reflexive, symmetric, transitive relation — think "generalized $\leq$") can express — ordering constraints among tasks.

- We'll look at one application — topological sorting. PERT charts discussed in book.

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Minute Essay

- None — quiz.

Slide 8