Slide 1

Administrivia

- Notice that my slides will be available linked from the “lecture topics and assignments” Web page, usually fairly soon after class. Usually there will be at least a preliminary version available before class as well. Answers to minute-essay questions (the ones that have answers) will be in the final version.
- It will probably help to bring the textbook to class most days.

Slide 2

Why Study Propositional Logic?

- Because it’s conceptually related to Boolean algebra (used in programming, circuit design, etc.).
- Because it’s related to proofs, which you should know a bit about.
- As an example of a “formal system” — represent something symbolically, define and apply rules for manipulating symbols, etc. Other examples in automata theory, theory of databases, etc.
- Because when you ask Dr. Theory (Myers) what you should learn in this course, he says “logic, logic, logic, logic!”
- Because after logic, the rest of the course will (probably) seem easy!

Slide 3

Propositional Logic — The Big Picture

- Underlying many fields is a notion of “valid argument”, one thing “following logically” from another — math, science, law, etc. (Consider example at the start of chapter 1.)
- Can define precisely what this means using natural language, but it’s difficult and clumsy.
- If we use mathematical notation instead, it’s easier to produce/follow chains of reasoning.

  (Analogous to “word problems” in algebra — the idea is to turn something that’s clumsy to work with into mathematical symbols, operate on the symbols with well-defined math, and translate the result back into words.)
- Emphasis in this course is on the logic/math part rather than on translating real-world English into symbols.

Slide 4

Statements / Propositions

- Definition — something (in natural language, a sentence) that is either true or false. (We might not know which.)
- Which of these are statements?
  - Water is wet.
  - Water is not wet.
  - Is the sky blue on Venus?
  - There is life on Mars.
- Notational convention — A, B, C, etc., are statements.

Slide 5

Connectives

- Can build up more complicated statements by combining simpler ones, using "connectives" — each has intuitive meaning, formal definition.
- "and" connective pretty clear — $A \land B$ defined by truth table
- "or" connective also pretty clear — $A \lor B$ defined by truth table

  Notice that this is "inclusive or" — not always the same as what we mean by "or" in natural language.
- "not" connective also pretty clear — A' defined by truth table
- "implies" connective is trickier — $A \rightarrow B$ defined by truth table

  Why define it that way? Stay tuned . . .
- "is equivalent to" connective also pretty clear — $A \leftrightarrow B$ defined by truth table

Slide 6

Why Did We Define Implication That Way?

- Definition of $A \rightarrow B$ when A is true seems reasonable, right?
- When A is false, though — why say $A \rightarrow B$ is true?
  - "Benefit of the doubt" argument: We have to call it either true or false, and it's not obviously false.
  - "It's math" argument: Maybe this definition *doesn't* express some fundamental truth. But in some sense math is its own universe, and we can define things any way we want (though we hope the definitions fit together in a nice way and maybe have applications).

    (In fact, some treatments of propositional logic just define implication formally, in terms of other connectives, and don't try to justify it.)

Slide 7

Compound Statements / Well-Formed Formulas

- Natural-language equivalents of statements joined by connectives:
  - Water is wet and grass is green.
  - If Jo(e) is a CS major, Jo(e) must take this course.
- We can "nest" connectives, e.g., $(A \wedge B)'$.
- We can define a notion of "well-formed formula" (wff) based on this (formal definition should be recursive, and we'll do that later) — basically, a "sensible" combination of statement letters, connectives, and parentheses.
- Notational convention — P, Q, . . . for wffs.
- We can use truth tables to figure out truth values for wffs. (How many rows do we need?) Let's do an example . . .

Slide 8

Compound Statements, Continued

- As an example, let's try turning the example at the start of chapter 1 into formulas, using the following:

  A is "The client is guilty."

  B is "The knife was in the drawer."

  C is "Jason P. saw the knife."

  D is "The knife was there on Oct. 10."

  E is "The hammer was in the barn."
- And then we hope we can somehow use the formulas to help us decide whether the conclusion follows from the premises.

Slide 9

More Definitions

- Some wffs are always true — "tautologies". Examples?
- Some wffs are always false — "contradictions". Examples?
- We can talk about two wffs P and Q being "equivalent" — $P \leftrightarrow Q$ is a tautology.

  Write $P \Leftrightarrow Q$.

  Table of common equivalences on p. 8.

  Additional widely-used equivalences — "De Morgan's Laws" (p. 9).

Slide 10

Propositional Formulas in Other Contexts

- Notice similarities between the connectives here and
  - Boolean expressions in programming languages.
  - Expressions for "advanced search" in some search engines, database queries, etc.
- Can use rules we have so far to simplify such expressions (e.g., De Morgan's Laws can be useful in simplifying boolean expressions in programs).

Slide 11

Propositional Logic — Valid Arguments

- Now we want to capture notion of "valid argument" — formal version of what someone familiar with proofs would recognize as such.
- Idea is that we have "hypotheses" $P_1, P_2, \ldots, P_n$ and "conclusion" Q, and we want to know when we can be sure that the truth of the hypotheses guarantees the truth of the conclusion — i.e., when is

$$(P_1 \wedge \ldots \wedge P_n) \rightarrow Q$$

a tautology?
- Could we use truth tables? If we can, would we always want to?

Slide 12

Valid Arguments, Continued

- A more algorithmic view — apply "derivation rules" to construct a "proof sequence".

  Idea is that we have a list of wffs that we know are true any time all the hypotheses $(P_1, P_2, \ldots P_n)$ are true. Then we proceed thus:

  1. Initialize this list to include just $P_1, P_2, \ldots P_n$.
  2. If conclusion Q is on the list, stop.
  3. Apply a derivation rule to one or more wffs in the list, producing a new wff X. Add X to the list.
  4. Go to step 2.

Slide 13

Derivation Rules

- What kind of "derivation rules" would be good?
  - When we apply one, we want it to be the case that if the wffs we start with are true, the wff we derive is also true — system is "sound". "Everything we can prove is true."
  - Together they are powerful enough to allow us to construct proof sequences for all true statements — system is "complete". "Everything that is true has a proof." (Possible here, but not for more complicated kinds of logic!)

Slide 14

Derivation Rules, Continued

- Two groups of basic rules:
  - Equivalence rules (two-way) — p. 24.
  - Inference rules (one-way) — p. 25.

  ("Do I have to memorize these?" No. Exams and quizzes will be open book.)
- Formally, these rules are true because we can prove them using truth tables.
- They should also seem plausible, maybe even "obviously true".
- Can derive additional rules; table on p. 33 lists some.

Slide 15

Building Blocks for Proof Sequences

- Equivalence rules (two-way), p. 24. Notice that these *can* be applied to parts of wffs.

  Example: "Implication" says that if we have $P \rightarrow Q$ we can replace it with $P' \lor Q$, or vice versa.
- Inference rules (one-way), p. 25. Notice that these *cannot* be applied to parts of wffs.

  Example: "Modus ponens" says if we have $P \rightarrow$ Q on one line, and P on another, we can write down a new line Q.
- "Deduction method": To show that $P_1, P_2, \ldots P_n$ guarantee conclusion $R \rightarrow Q$, we can show that $P_1, P_2, \ldots P_n, R$ guarantee Q
- Derived inference rules, p. 33. Notice that many of these are proved as problems, and you should only use them for later problems. (E.g., okay to use the results of problem 23 in problem 25, but not vice versa.)

Slide 16

Proof Sequences — Simple Example

- So, let's do a trivial example:
  - Hypotheses:

    P

    $P \rightarrow Q$
  - Conclusion:

    Q
- "Justifications" we write down for each step aren't technically required for a valid proof sequence. We put them in to help human readers.

  (Be aware that this isn't the only format for doing such proofs. Different books/authors use different formats. Same ideas behind all of them, though.)

Slide 17

Minute Essay

- Suppose we have
  - W is “Water is wet.”
  - L is “There is life on Mars.”

  Write as wffs the following:
  - “Water is wet and there is life on Mars.”
  - “There is life on Mars if water is wet.”

Slide 18

Minute Essay Answer

- “Water is wet and there is life on Mars.”:

  $W \wedge L$
- “There is life on Mars if water is wet.”:

  $W \rightarrow L$