Slide 1

Administrivia

- Homework 1 on the Web; due a week from today (5pm).

Slide 2

Minute Essay From Last Lecture

- (Almost everyone got it right. Good!)

Slide 3

Propositional Logic — Valid Arguments (Review)

- Now we want to capture notion of "valid argument" — formal version of what someone familiar with proofs would recognize as such.
- Idea is that we have "hypotheses" $P_1, P_2, \ldots, P_n$ and "conclusion" Q, and we want to know when we can be sure that the truth of the hypotheses guarantees the truth of the conclusion — i.e., when is

$$(P_1 \ \wedge \ \ldots \ \wedge \ P_n) \ \rightarrow \ Q$$

  a tautology?
- We could we use truth tables, but it's not clear we would always want to.

Slide 4

Valid Arguments, Continued (Review)

- A more algorithmic view — apply "derivation rules" to construct a "proof sequence".

  Idea is that we have a list of wffs that we know are true any time all the hypotheses $(P_1, P_2, \ldots P_n)$ are true. Then we proceed thus:

  1. Initialize this list to include just $P_1, P_2, \ldots P_n$.
  2. If conclusion Q is on the list, stop.
  3. Apply a derivation rule to one or more wffs in the list, producing a new wff X. Add X to the list.
  4. Go to step **??**.

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Building Blocks for Proof Sequences

- Equivalence rules (two-way), p. 24. Notice that these *can* be applied to parts of wffs.

  Example: "Implication" says that if we have $P \rightarrow Q$ we can replace it with $P' \vee Q$, or vice versa.

- Inference rules (one-way), p. 25. Notice that these *cannot* be applied to parts of wffs.

  Example: "Modus ponens" says if we have $P \rightarrow$ Q on one line, and P on another, we can write down a new line Q.

- "Deduction method": To show that $P_1, P_2, \ldots P_n$ guarantee conclusion $R \rightarrow Q$, we can show that $P_1, P_2, \ldots P_n, R$ guarantee Q

- Derived inference rules, p. 33. Notice that many of these are proved as problems, and you should only use them for later problems. (E.g., okay to use the results of problem 23 in problem 25, but not vice versa.)

Slide 6

Proof Sequences — Simple Example

- So, let's do an example:
  - Hypotheses:

    A

    B
  - Conclusion:

    $(A \wedge B) \vee C$

- "Justifications" we write down for each step aren't technically required for a valid proof sequence. We put them in to help human readers.

  (Be aware that this isn't the only format for doing such proofs. Different books/authors use different formats. Same ideas behind all of them, though.)

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Hints for Constructing Proof Sequences

- Two things involved in constructing proofs:
  - Applying the rules correctly — not so difficult, if you correctly match up your formula with the rule.
  - Knowing which rule to apply — more difficult, gets a little easier with practice. Also see hints (“heuristics”) on p. 27:
    - Consider using modus ponens often.
    - Consider using De Morgan’s laws to simplify (?) $(P \vee Q)'$, $(P \wedge Q)'$.
    - Consider using equivalence rules to convert $P \vee Q$ to $P' \rightarrow Q$.

    Sometimes helps to “work backward” — figure out an intermediate result from which you could reach the conclusion, then figure out how to get the intermediate result. If you do this, though, must still construct proof “going forward”.

Slide 8

Example(s)

- Lawyer example from start of chapter.
- Section 1.2 problems 19, 20, 26, 35, 40, 46. (Divide into groups and do at board.)

Slide 9

So, What Does This Buy Us?

- Yes, this can seem long and tedious. But . . .
- It's in some ways easier than other approaches, and certainly more reliable.
- Compare to "word problems" in algebra — first convert from natural language to math, apply math, convert back — with practice, easier and more reliable than guessing.
- In a way, we're replacing thinking with symbol manipulation!

Slide 10

Minute Essay

- How much of this (if any) looks familiar to you from other courses?