Slide 1

Administrivia

- Reminder: Quiz 1 Thursday. Okay to use textbook and any notes. Questions will be about propositional logic.
- One-page summaries of rules for propositional and predicate logic on Web ("Useful links and other resources" page). Somewhat redundant given Appendix A.
- Reminder: Homework 1 due today.
- Homework 2 should be on the Web later today. Due in a week.
- (Review minute essay from last time.)

  In minute essays you can also ask any questions that occur to you about the class or related subjects, and I'll try to answer.)
- Note that we will skip 1.6 for now, and 1.5 completely.

Slide 2

Valid Arguments, Revisited — Review

- As with propositional logic, we want to know when we can say that a conclusion "logically follows" from a set of hypotheses — i.e., no matter what interpretation we choose, if the hypotheses are true so is the conclusion.
- What we have in our "bag of tricks":
  - All propositional-logic rules.
  - New rules for manipulating quantifiers.

Slide 3

Universal Instantiation — Review

- Rule for removing $\forall$. (Why do we want to do this?)
- If we have $(\forall x)P(x)$

  we can write $P(t)$

  provided t doesn't already exist "bound" in $P(x)$.
- "If $P(x)$ for all x, then $P(t)$ for a particular t".

Slide 4

Existential Instantiation — Review

- Rule for removing $\exists$. (Why do we want to do this?)
- If we have $(\exists x)P(x)$

  we can write $P(t)$

  provided t has not been previously used in the proof.
- "If there is some x for which $P(x)$, we can give it a name — t, for example."

Slide 5

Universal Generalization — Review

- Rule for introducing $\forall$. (Why do we want to do this?)
- If we have $P(x)$

  we can write $(\forall x)P(x)$

  provided x is "arbitrary" — not a free variable in a hypothesis, not a variable we got from ei, not a free variable in a formula we got from ei. (For last part, consider example at bottom of p. 49.)

  (Yes, this is tricky to understand/apply.)
- "If we know $P(x)$ for arbitrary x, then $P(x)$ for all x."

Slide 6

Existential Generalization — Review

- Rule for introducing $\exists$. (Why do we want to do this?)
- If we have $P(y)$ or $P(a)$

  we can write $(\exists x)P(x)$

  provided x doesn't appear in $P(a)$.
- "If we have some particular z for which $P(z)$, then there exists a z such that $P(z)$."

Slide 7

Predicate Logic, Recap / What Next?

- Now we have a set of derivation rules for predicate logic (we'll add a few more for convenience later).
- As with propositional logic, we could show that these rules are "sound" (if we can prove something, it's true/valid) and "complete" (if something is true/valid, we can prove it).

Slide 8

Temporary Hypotheses

- In propositional logic, we allowed proving a conclusion of the form $P \rightarrow Q$ by adding P to the list of hypotheses and proving Q.
- Along the same lines, we allow "temporary hypotheses":
  Suppose as part of a proof we want to show that $R \rightarrow S$ follows from the hypotheses. If $R \rightarrow S$ is the conclusion, deduction method works. What if it's not? Then we can't just add R to the list of hypotheses. What to do?
- One solution would be (in mathspeak) a lemma ("branch" or side proof).

Temporary Hypotheses, Continued

- Another solution is basically an inline lemma:
  - Introduce "temporary hypothesis" T.
  - Derive some more steps from earlier results and T, ending with S.
  - Conclude that $T \rightarrow S$.

  Note that the formulas we derive from earlier steps and T might depend on T, so — indent to make it clear that they're not part of the main proof.
- Example — section 1.4 problem 22.

Slide 9

One More Rule, a Conclusion

- One more rule — negation (example 32 p. 56).
- A conclusion — the goal of formal logic is to make arguments as meaningless as possible (!) — i.e., abstract out everything that doesn't matter, and apply formal mathematical rules to what's left.

Slide 10

Predicate Logic — Proof Sequence Sketch

- Start by removing quantifiers (with ei, ui rules) — usually remove existential quantifiers first, then universal.
- Apply rules from propositional logic to get unquantified result.
- Use eg, ug rules to put quantifiers back in.

Slide 11

More Examples

- Translating English to formulas: Section 1.3 problem 11.
- “Word problem”: Section 1.4 problem 34.

Slide 12

Minute Essay

- What (if anything!) did you find interesting about Homework 1?
- What (if anything!) did you find difficult about Homework 1?

Slide 13