Slide 1

Administrivia

- (None.)

Slide 2

Counting (Combinatorics)

- “Counting” sounds too trivial for a college-level course, right? but consider situations in which you want to know how many things there are in a set but don’t actually want to list them all. We will look at several ways to “count” without actually enumerating.
- (But, but, if you know how to write programs, why not just actually count . . . ? or could there still be too many to feasibly list?)

Slide 3

Multiplication Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2, how many outcomes are there for the sequence "event 1, then event 2"?
- Pictorially — draw a tree. Clear that there are $N_1 \times N_2$.
- Can easily extend by induction to sequences of more than two events.
- Example: If a password consists of 4 decimal digits, how many are there? (And if we allowed 10 seconds to try each one, how long would it take to try them all?)
- Example: If a license-plate number is 3 decimal digits followed by three alphabetic characters, how many are there?

Slide 4

Addition Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2 (and the sets of "event 1 outcomes" and "event 2 outcomes" are disjoint), how many outcomes are there for the event "event 1 or event 2"?
- Fairly easy to see that there are $N_1 + N_2$ possibilities in all.
- Can also easily extend by induction to combinations of more than two events.
- Example: If you have to choose an elective from either the Department of Esoteric Pursuits (which offers 10 of them) or from the Department of Life Skills (which offers 6 of them), how many choices are there in all (assuming no courses are cross-listed)?

Combining the Addition and Multiplication Principles

- Example: How many phone numbers are there that have either area code 210 or area code 512?
- Example: How many 7-digit phone numbers have at least one repeated digit?

Slide 5

Decision Trees

- Sometimes it's useful to represent a sequence of choices as a "decision tree" and explicitly count leaf nodes.
- Example: How many ways are there to get 4 coin tosses with no sequences of three heads or three tails?

Slide 6

More Examples

- Section 3.2 problems 45, 46, 60.

Slide 7

Principle of Inclusion/Exclusion

- Motivating(?) example:

  You take a poll of how many people support propositions A and B. You find that 10 of them support A, 20 support B, and 5 support both A and B. How many support either A or B?

- Using set notation, with $|S|$ meaning the number of elements in S:
  Given $|A| = 10$, $|B| = 20$, and $|A \cap B| = 5$,
  what is $|A \cup B|$?

- We can use the addition principle to derive

$$|A \cup B| = |A| + |B| - |A \cap B|$$

  (Intuitive idea is that we count everything in both sets, and in doing that we count some things twice, so we must correct.)

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Slide 9

Principle of Inclusion/Exclusion, Continued

- What if there were three propositions/sets? Can we derive a rule?
- Sure . . . (next slide).

Slide 10

Principle of Inclusion/Exclusion, Continued

- Rule for three sets is

  $$|A \cup B \cup C| = |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$$

- Intuitive idea:

  Count all the A's, all the B's, all the C's.

  A&B's, B&C's, and A&C's have been counted twice; A&B&C's have been counted three times.

  Subtract counts of A&B's, B&C's, and A&C's; now A&B&C's have been counted zero times.

  Add count of A&B&C's.

- Formally, derive from rule for two sets and rules for set operations.

Principle of Inclusion/Exclusion, Continued

- There's a pattern, captured in general form of rule (p. 228). (In another textbook — "A Ghastly Formula".)
- For more interesting examples (most beyond the scope of this course), Google "inclusion/exclusion principle".

Slide 11

Pigeonhole Principle

- Idea is that if you have n items placed in k bins, and $n > k$, then at least one bin has more than one item.

  Converse is that if no bin contains more than one item, n can be at most — what?

  More general version — if you have k bins and more than mk items, there's at least one bin with more than m items.
- Example — section 3.3 problem 22.

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Slide 13

Pigeonhole Principle, Continued

- Another example (discovered on a Web page at Stanford, no longer available):

  If A is a set of 10 integers in the range 1 to 100, show that there are at least two distinct and disjoint subsets of A that have the same sum.

  (Idea is to count number of possible subsets and also figure out range of potential sums. If more subsets than possible sums . . .)

Slide 14

Permutations

- We might want to know how many ways we can choose an ordered sequence of r objects, chosen from n possibilities with no repeats. Call this $P(n, r)$.

  Example: How many 7-digit phone numbers have no repeated digits?

- Can we come up with a general formula? (Of course. Let's derive one.)

- Look at some boundary cases — $r = n$, $r = 0$, $r = 1$, etc. (We'll need to agree that $0! = 1$.)

Slide 15

Combinations

- Or we might want to know how many ways we can choose an *unordered* collection of r objects, chosen from n possibilities with no repeats. Call this $C(n, r)$.

  Example: How many ways can we draw 5 cards from a deck of 52?
- Can we come up with a general formula? (Of course. Let's derive one.)
- Again look at some boundary cases — $r = n$, $r = 1$, $r = 0$.
- (Another common notation for this is $\binom{n}{r}$ ("n choose r").)

Slide 16

Minute Essay

- None — quiz.