Slide 1

Administrivia

- (None.)

Slide 2

Permutations and Combinations — Review/Recap

- How many ways to choose an ordered sequence of r objects, chosen from n possibilities with no repeats?
  $P(n, r) = n!/(n - r)!$
- How many ways to choose an unordered sequence of r objects, chosen from n possibilities with no repeats?
  $C(n, r) = n!/((n - r)!r!)$

Permutations Versus Combinations

- In general: If order matters, it's a permutation; if order doesn't matter, it's a combination.
- (Contrast "how many phone numbers with no repeated digits" (order matters) with "how many 5-card hands?" (order doesn't matter).)

Slide 3

Potential Pitfall — Counting Things Twice

- A problem is that some proposed solutions sound reasonable but actually manage to count some things twice, or don't count some things at all.
- Example: example 55 part (d).

Slide 4

Slide 5

Permutations and Combinations — Eliminating Duplicates

- In general it can be (somewhat) interesting to try to figure out how to "eliminate duplicates" — i.e., account for the fact that one way of counting things produces a lot of duplicate results.
- Example: How many ways can we rearrange the letters in the word "voodoo"?

Slide 6

Permutations and Combinations With Repetitions

- Definitions of $P(n, r)$ and $C(n, r)$ specified "without repeats". What if we want to allow repeats?
- For permutations, not too tough — n^r ways to choose an ordered sequence of r things from n possibilities, if we allow repeats?
- For combinations, it's trickier. How many ways can we choose an unordered collection of r things from n possibilities, if we allow repeats? Use a clever idea from example 58.

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Permutations and Combinations, Example(s)

- How many ways to draw 5 cards from a standard 52-card deck and get three of a kind and a pair?

Slide 8

Probability — Equally-Likely Outcomes

- Basic definition: If S ("sample space") is a set of equally likely outcomes of some action (e.g., possible results of tossing a fair coin), and E ("event") is a subset of S, then we define the probability of E as

$$P(E) = \frac{|E|}{|S|}$$

  Examples: Sequences of coin tosses, 5-card "hands" chosen from 52-card deck, etc.

- Note that $0 \leq P(E) \leq 1$. (Why?) When is $P(E) = 0$? When is $P(E) = 1$?

- Note that we can apply anything we know about sizes of sets. (E.g., if E_1 and E_2 are disjoint, what is $P(E_1 \cup E_2)$ in terms of $P(E_1)$ and $P(E_2)$?)

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Example(s)

- In a group of n people, what's the probability that at least two people have the same birthday?

Slide 10

Minute Essay

- Given 20 words, how many 6-word phrases can you make up, if no repeated words are allowed? ("refrigerator magnet poetry")
  Okay to express answers in terms of $P(n, r)$ and/or $C(n, r)$ or factorials.
- Suppose you select 6 marbles at random from a jar containing red, blue, yellow, and green marbles (at least 6 each). How many ways can this selection be made?

Minute Essay Answer

- Order matters here, so $P(20, 6)$
- Order doesn't matter here, but repetitions are allowed, so this is a case of "combinations with repetitions", so there are $C(6 + 4 - 1, 6)$ (=$C(9, 6)$) ways to select.

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