**Slide 1**

## Administrivia

- Homework 1 on the Web; due a week from today (5pm).
- Notice that my slides will be available linked from the "lecture topics and assignments" Web page, usually fairly soon after class. Usually there will be at least a preliminary version available before class as well. Answers to minute-essay questions (the ones that have answers) will be in the final version.
- It will probably help to bring the textbook to class most days.

**Slide 2**

## Propositional Logic — Review/Recap

- Last time we defined some terms for building up formulas that represent things that can be true/false — statements, connectives, etc.
- As an example, let's try turning the example at the start of chapter 1 into formulas, using the following:

  $A$ is "The client is guilty."

  $B$ is "The knife was in the drawer."

  $C$ is "Jason P. saw the knife."

  $D$ is "The knife was there on Oct. 10."

  $E$ is "The hammer was in the barn."
- And then we hope we can somehow use the formulas to help us decide whether the conclusion follows from the premises.

**Slide 3**

## Propositional Logic — Valid Arguments

- Now we want to capture notion of "valid argument" — formal version of what someone familiar with proofs would recognize as such.
- Idea is that we have "hypotheses" $P_1, P_2, \ldots, P_n$ and "conclusion" $Q$, and we want to know when we can be sure that the truth of the hypotheses guarantees the truth of the conclusion — i.e., when is

$$(P_1 \ \wedge \ \ldots \ \wedge \ P_n) \ \rightarrow \ Q$$

  a tautology?
- Could we use truth tables? If we can, would we always want to?

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## Valid Arguments, Continued

- A more algorithmic view — apply "derivation rules" to construct a "proof sequence".

  Idea is that we have a list of wffs that we know are true any time all the hypotheses $(P_1, P_2, \ldots P_n)$ are true. Then we proceed thus:

  1. Initialize this list to include just $P_1, P_2, \ldots P_n$.
  2. If conclusion $Q$ is on the list, stop.
  3. Apply a derivation rule to one or more wffs in the list, producing a new wff $X$. Add $X$ to the list.
  4. Go to step 2.

## Derivation Rules

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- What kind of “derivation rules” would be good?
  - When we apply one, we want it to be the case that if the wffs we start with are true, the wff we derive is also true — system is “sound”. “Everything we can prove is true.”
  - Together they are powerful enough to allow us to construct proof sequences for all true statements — system is “complete”. “Everything that is true has a proof.” (Possible here, but not for more complicated kinds of logic!)

## Derivation Rules, Continued

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- Two groups of basic rules:
  - Equivalence rules (two-way) — p. 24.
  - Inference rules (one-way) — p. 25.

  (“Do I have to memorize these?” No. Exams and quizzes will be open book.)
- Formally, these rules are true because we can prove them using truth tables.
- They should also seem plausible, maybe even “obviously true”.
- Can derive additional rules; table on p. 33 lists some.

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## Building Blocks for Proof Sequences

- Equivalence rules (two-way), p. 24. Notice that these *can* be applied to parts of wffs.

  Example: "Implication" says that if we have $P \rightarrow Q$ we can replace it with $P' \vee Q$, or vice versa.

- Inference rules (one-way), p. 25. Notice that these *cannot* be applied to parts of wffs.

  Example: "Modus ponens" says if we have $P \rightarrow$ Q on one line, and $P$ on another, we can write down a new line $Q$.

- "Deduction method": To show that $P_1, P_2, \ldots P_n$ guarantee conclusion $R \rightarrow Q$, we can show that $P_1, P_2, \ldots P_n, R$ guarantee $Q$

- Derived inference rules, p. 33. Notice that many of these are proved as problems, and you should only use them for later problems. (E.g., okay to use the results of problem 23 in problem 25, but not vice versa.)

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## Proof Sequences — Simple Example

- So, let's do a trivial example:
  - Hypotheses:

    $P$

    $P \rightarrow Q$
  - Conclusion:

    $Q$

- "Justifications" we write down for each step aren't technically required for a valid proof sequence. We put them in to help human readers.

  (Be aware that this isn't the only format for doing such proofs. Different books/authors use different formats. Same ideas behind all of them, though.)

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## Hints for Constructing Proof Sequences

- Two things involved in constructing proofs:
  - Applying the rules correctly — not so difficult, if you correctly match up your formula with the rule.
  - Knowing which rule to apply — more difficult, gets a little easier with practice. Also see hints (“heuristics”) on p. 27:
    - Consider using modus ponens often.
    - Consider using De Morgan’s laws to simplify (?) $(P \vee Q)'$, $(P \wedge Q)'$.
    - Consider using equivalence rules to convert $P \vee Q$ to $P' \rightarrow Q$.

    Sometimes helps to “work backward” — figure out an intermediate result from which you could reach the conclusion, then figure out how to get the intermediate result. If you do this, though, must still construct proof “going forward”.

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## Example(s)

- Lawyer example from start of chapter.

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## So, What Does This Buy Us?

- Yes, this can seem long and tedious. But . . .
- It's in some ways easier than other approaches, and certainly more reliable.
- Compare to "word problems" in algebra — first convert from natural language to math, apply math, convert back — with practice, easier and more reliable than guessing.
- In a way, we're replacing thinking with symbol manipulation!

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## Minute Essay

- Suppose we have
  - $W$ is "Water is wet."
  - $L$ is "There is life on Mars."

  Write as wffs the following:
  - "Water is wet and there is life on Mars."
  - "There is life on Mars if water is wet."
- How much of this (if any) looks familiar to you from other courses?

## Minute Essay Answer

- "Water is wet and there is life on Mars.":

  $W \ \wedge \ L$

- "There is life on Mars if water is wet.":

  $W \ \rightarrow \ L$

**Slide 13**