Slide 1

Administrivia

- Reminder: Homework 3 due Tuesday.
- Reminder(?): Midterm coming up soonish — March 5. I would like to include material from today and next time. So . . .
- Quiz 3 a week from today. Homework 4 on the Web. Officially due the day of the exam, but I will prepare solutions early.

Slide 2

Mathematical Induction — Review/Recap

- Questions usually phrased as "prove that $P(n)$ is true for all integers $\geq n_0$", where $P(n)$ is some statement about n (equation, not formula).
- Two "proof obligations":
  - Base case — usually just n_0, but sometimes must include few numbers right after n_0 as well. (e.g., Example 24 in section 2.2).
  - Inductive step. Notice that *what you are proving is an implication*.
- Why this works — you are proving base cases and a rule for constructing implications, after which you can use universal instantiation and *modus ponens* to get results for non-base cases.

Slide 3

Mathematical Induction — Inductive Step Hints

- What generally works, assuming inductive hypothesis is equation ($f(k) = g(k)$):
  - Write down one side of equation to be proved ($f(k+1)$).
  - Rewrite it so it somehow includes $f(k)$.
  - Replace $f(k)$ with $g(k)$, then do algebra to show the whole expression equals $g(k+1)$.
- If proving an inequality, often helpful to use the fact that if $x \leq y$ and $y \leq z$, then $x \leq z$.

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More Examples

- Section 2.2 problems 31 (finish), 64. (We were meant to notice, for problem 31, a previous problem — 28. But we could do the problem without that.)

Slide 5

Recursion and Recursive Definitions

- Idea of recursion closely related to idea of induction — “build on previous smaller cases”.
- First look at recursive definitions. To define something recursively:
  - Define one or more “base cases”.
  - Define remaining cases in terms of other (“smaller”) cases.

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Recursive Definitions — Sequences

- A silly example:

$$\begin{aligned} S(1) &= 1 \\ S(n) &= S(n-1) \times 10, \quad \text{for } n > 1 \end{aligned}$$

  Try writing down some terms.

- Another example:

$$\begin{aligned} S(1) &= 1 \\ S(2) &= 1 \\ S(n) &= S(n-2) + S(n-1), \quad \text{for } n > 2 \end{aligned}$$

  Try writing down some terms. Anyone recognize this one?

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Recursive Definitions — Sets

- Example — could define the set of "integer arithmetic expressions" like this:
  - Integers are expressions.
  - If E and F are integer arithmetic expressions, so are $(E+F)$, $(E-F)$, $(E \times F)$, and (E/F).

  Examples?

  Notice that this allows us to generate only "sensible" expressions. Notice also that it's a bit more restrictive than we might like.
- We could write similar definitions for the wffs of propositional and predicate logic.
- Notice: To claim that something is in the set you need to be able to show that it's either a base case or can be obtained from a base case by applying one of the "rules" that define the set.

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Recursive Definitions — Operations

- Example — factorial.
- Example — multiplication of non-negative integers, defined in terms of addition.
- Example — (integer) division of a non-negative integer by a positive integer, defined in terms of subtraction.

Minute Essay

- Prove using mathematical induction that for all $n \geq 1$

$$\sum_{i=1}^{n}(2i-1) = n^2$$

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Minute Essay Answer

- Base case: $n = 1$. $n^2 = 1$ and

$$\sum_{i=1}^{n}(2i-1) = 1$$

- Inductive step: Assume

$$\sum_{i=1}^{k}(2i-1) = k^2$$

and show

$$\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$$

Using inductive hypothesis:

$$\sum_{i=1}^{k+1}(2i-1) = \sum_{i=1}^{k}(2i-1) + 2(k+1) - 1 = k^2 + 2k + 1 = (k+1)^2$$

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