Slide 1

Administrivia

- Reminder: Homework 5 due today. (Accepted without penalty through Friday at 5pm.)

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Sets

- (This will likely be review for most of you!)
- Definition: Informally, a set is a collection of objects (unordered, no duplicates). Formally — well, formal definitions are surprisingly difficult! (Skim the Wikipedia article “Russell’s paradox” for a bit more information.)
- Some notation — for x an object and A a set,
  $x \in A$ means — ?
  $y \notin A$ means — ?
- We say two sets are equal exactly when they have the same members.

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Ways to Specify Sets

- By listing elements, e.g., $S = \{a, b, 1, 2\}$.
- Recursively, as in chapter 2.
- By describing a property P such that x is in S exactly when $P(x)$. E.g., $S = \{x \mid x \text{ is an even integer}\}$
- As one of
  - $\{\}$ or $\emptyset$ (empty set).
  - $\mathbb{N}$ (non-negative integers).
  - $\mathbb{Z}$ (integers).
  - $\mathbb{Q}$ (rationals).
  - $\mathbb{R}$ (reals).
  - $\mathbb{C}$ (complex numbers).

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Subsets

- $A \subseteq B$ exactly when every element of A is also in B. "Proper" subset is when $A \neq B$.

  For what sets S is the empty set a subset of S?
- If $A \subseteq B$ and $B \subseteq A$, what do we know about A and B?

Power Sets

- Sets are collections of objects, so no reason we can't have sets of sets, right?
- For set S, define $\mathscr{P}(S)$ ("power set of S") to be the set of all subsets of S.
- If S is finite and has n elements, how many elements in $\mathscr{P}(S)$? (See textbook for nice inductive proof.)

Slide 5

Operations on Sets

- Union: $A \cup B = \{x \mid x \in A \ \vee \ x \in B\}$.
- Intersection: $A \cap B = \{x \mid x \in A \ \wedge \ x \in B\}$. What does "$A$ and B are disjoint" mean?
- Complement: $A' = \{x \mid x \in S \ \wedge \ x \notin A\}$, where S is some "universal set" (without which this definition doesn't make sense) — integers, people, etc.
- Difference: $A - B = \{x \mid x \in A \ \wedge \ x \notin B\}$.
- Cartesian product: $A \times B = \{(x, y) \mid x \in A \ \wedge \ y \in B\}$.

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Properties of Set Operations

- These operations have many useful properties — commutativity, associativity, etc. — see p. 171 for a list.
- All of these properties can be proved from the definition ($A = B$ exactly when $A \subseteq B$ and $B \subseteq A$). Example — show $A \cup B = B \cup A$.

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Countable and Uncountable Sets

- If A and B are finite sets, fairly obvious what it means for them to be "the same size", right?
- Is there some way to extend this to notion of "size" for infinite sets?

Countable and Uncountable Sets, Continued

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- A bit informally, we can say that two sets are the same size ("have the same cardinality") if we can set up a one-to-one correspondence between them.
- For finite sets, matches our earlier/intuitive ideas, right? How about infinite sets?
  - Positive integers versus negative integers?
  - Even integers versus odd integers?
  - Integers versus even integers?

Countable and Uncountable Sets, Continued

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- Define "S countable" to mean there's some way to write down all elements of S "in order". (Might be more than one way — okay so long as there's at least one.)
- Are the following sets countable?
  - Finite sets?
  - $\mathbb{N}$?
  - $\mathbb{Z}$?
  - $\mathbb{Q}^+$?

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Countable and Uncountable Sets, Continued

- So are all sets countable?? No. $\mathbb{R}$ is not.

  Proof is by contradiction. First we notice that we can set up a one-to-one correspondence between all real numbers and the real numbers greater than 0 and less than 1. Then we assume we can "list" those numbers and show that there's one we missed.
- Is $\mathbb{R}$ the "largest" set? No. We can also prove that S and $\mathscr{P}(S)$ are not "the same size", again by contradiction. ("Cantor's theorem")
- (Is any of this crucially important to an understanding of computer science? Probably not, but it's too entertaining to skip.)

Slide 12

Minute Essay

- Suppose you have

  $A = \{2, 4, 6, 8\}$

  $B = \{1, 4, 9, 16\}$

  What are $A \cup B$, $A \cap B$, and $A - B$? How many elements are there in $\mathscr{P}(A)$?

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Minute Essay Answer

- $A \cup B = \{1, 2, 4, 6, 8, 9, 16\}$
- $A \cap B = \{4\}$
- $A - B = \{2, 6, 8\}$?
- There are 4 elements in A, so there are 16 (2^4) elements in $\mathscr{P}(A)$.