Slide 1

Administrivia

- Reminder: Quiz 4 Thursday. Likely topics are recurrence relations, analysis of algorithms.
- Next homework (problems from chapter 3) coming soon; to be due a week from Thursday.

Slide 2

Counting (Combinatorics)

- "Counting" sounds too trivial for a college-level course, right? but consider situations in which you want to know how many things there are in a set but don't actually want to list them all.
  - Given what a password is supposed to look like (4 digits, 20 characters, etc.), how many are there? i.e., how easy would it be to guess?
  - Given a scheme for IP addresses, how many are possible? i.e., are there enough for everything we want to give one to?

  We will look at several ways to "count" without actually enumerating.
- (But, but, if you know how to write programs, why not just actually count . . . ? or could there still be too many to feasibly list?)

Slide 3

Multiplication Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2, how many outcomes are there for the sequence “event 1, then event 2”?
- Pictorially — draw a tree. Clear that there are $N_1 \times N_2$.
- Can easily extend by induction to sequences of more than two events.
- Example: If a password consists of 4 decimal digits, how many are there? (And if we allowed 10 seconds to try each one, how long would it take to try them all?)
- Example: If a license-plate number is 3 decimal digits followed by three alphabetic characters, how many are there?

Slide 4

Addition Principle

- If there are N_1 outcomes for event 1 and N_2 outcomes for event 2 (and the sets of “event 1 outcomes” and “event 2 outcomes” are disjoint), how many outcomes are there for the event “event 1 or event 2”?
- Fairly easy to see that there are $N_1 + N_2$ possibilities in all.
- Can also easily extend by induction to combinations of more than two events.
- Example: If you have to choose an elective from either the Department of Esoteric Pursuits (which offers 10 of them) or from the Department of Life Skills (which offers 6 of them), how many choices are there in all (assuming no courses are cross-listed)?

Combining the Addition and Multiplication Principles

- Example: How many phone numbers are there that have either area code 210 or area code 512?
- Example: How many 7-digit phone numbers have at least one repeated digit?

Slide 5

Decision Trees

- Sometimes it's useful to represent a sequence of choices as a "decision tree" and explicitly count leaf nodes.
- Example: How many ways are there to get 4 coin tosses with no sequences of three heads or three tails?

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More Examples

- Section 3.2 problems 45, 46, 60.

Slide 7

Principle of Inclusion/Exclusion

- Motivating(?) example:

  You take a poll of how many people support propositions A and B. You find that 10 of them support A, 20 support B, and 5 support both A and B. How many support either A or B?

- Using set notation, with $|S|$ meaning the number of elements in S:
  Given $|A| = 10$, $|B| = 20$, and $|A \cap B| = 5$,
  what is $|A \cup B|$?

- We can use the addition principle to derive

$$|A \cup B| = |A| + |B| - |A \cap B|$$

  (Intuitive idea is that we count everything in both sets, and in doing that we count some things twice, so we must correct.)

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Principle of Inclusion/Exclusion, Continued

- What if there were three propositions/sets? Can we derive a rule?
- Sure . . . (next slide).

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Principle of Inclusion/Exclusion, Continued

- Rule for three sets is

  $$|A \cup B \cup C| = |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$$

- Intuitive idea:

  Count all the A's, all the B's, all the C's.

  A&B's, B&C's, and A&C's have been counted twice; A&B&C's have been counted three times.

  Subtract counts of A&B's, B&C's, and A&C's; now A&B&C's have been counted zero times.

  Add count of A&B&C's.

- Formally, derive from rule for two sets and rules for set operations.

Principle of Inclusion/Exclusion, Continued

- There's a pattern, captured in general form of rule (p. 228). (In another textbook — "A Ghastly Formula".)
- For more interesting examples (most beyond the scope of this course), Google "inclusion/exclusion principle".

Slide 11

Pigeonhole Principle

- Idea is that if you have n items placed in k bins, and $n > k$, then at least one bin has more than one item.

  Converse is that if no bin contains more than one item, n can be at most — what?

  More general version — if you have k bins and more than mk items, there's at least one bin with more than m items.
- Example — section 3.3 problem 22.

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Pigeonhole Principle, Continued

- Another example (discovered on a Web page at Stanford, no longer available):

  If A is a set of 10 integers in the range 1 to 100, show that there are at least two distinct and disjoint subsets of A that have the same sum.

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Pigeonhole Principle, Continued

- Example, continued: Idea is to count number of possible subsets and also figure out range of potential sums.

  If more subsets than possible sums . . .

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Slide 15

Minute Essay

- None — sign in.