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Administrivia

- As announced by e-mail, Reading Quiz 1 and Homework 1 graded; sample solutions and graded work on Google Drive.
- Reminder: Reading Quiz 2 due today.
- Homework 2 posted. Due next Monday.
- A few words about the textbook: If you're finding it sometimes slow going, *so am I*, alas. One online review I read said maybe this was not a good beginner book, despite its many strengths. I'm inclined to agree!

Slide 2

Asymptotic Notation and Proofs, Revisited

- In Homework 2 I ask you to prove Theorem 3.1 in the textbook. So, an example of proving things using the definitions of asymptotic notation:
- Exercise 3.2-5 in the textbook asks you to prove something I thought was interesting . . .

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Proof Example

- The problem as stated:

  Prove that the running time of an algorithm is $\Theta(g(n))$ if and only if its worst-case running time is $O(g(n))$ and its best-case running time is $\Omega(g(n))$.

- When proving something of the form "A if and only if B", we need to prove two things:
  - If A then B.
  - If B then A.

  See how far we get just writing down relevant definitions ...

- Some notation first: Use $f(n)$ to mean running time of the algorithm, $f_w(n)$ to mean its worst-case running time, and $f_b(n)$ to mean its best-case running time.

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Proof Example, Continued

- First show:

  If $f(n) = \Theta(g(n))$, then $f_w(n) = O(g(n))$ and $f_b(n) = \Omega(g(n))$.

- From the definition of Θ, there are positive constants c_1, c_2, and n_0 such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$.

- This has to be true for all cases, including the worst and best cases, so:

  $0 \leq f_w(n) \leq c_2 g(n)$ for all $n \geq n_0$, which is the definition of $O(g(n))$.

  and $0 \leq c_1 g(n) \leq f_b(n)$ for all $n \geq n_0$, which is the definition of $\Omega(g(n))$.

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Proof Example, Continued

- Now show:

  If $f_w(n) = O(g(n))$ and its $f_b(n) = \Omega(g(n))$, then $f(n) = \Theta(g(n))$.

- From the definitions (doing a bit of renaming of constants):

  There are positive constants c_2 and n_2 such that $0 \leq f_w(n) \leq c_2 g(n)$ for all $n \geq n_2$.

  There are positive constants c_1 and n_1 such that $0 \leq c_1 g(n) \leq f_b(n))$ for all $n \geq n_1$.

  Note that $f_b(n) \leq f(n) \leq f_w(n)$, so if we let n_0 be the larger of n_1 and n_2, for all $n \geq n_0$,

$$0 \leq c_1 g(n) \leq f_b(n) \leq f(n) \leq f_w(n) \leq c_2 g(n)$$

  which is pretty much the definition of $f(n) = \Theta(g(n))$

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Proof Example, Continued

- Done!
- Note, maybe, that this problem yielded to my usual approach for doing proofs: Start by writing down what you know, and see where you can go from there.

Quicksort

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- You've probably heard of quicksort as a sorting algorithm? I think it's useful in this course as an example of several things: It's an example of divide and conquer (and one where the analysis of run time is a little tricky), and one where the "split" part is nontrivial and I think benefits from an approach based on a loop invariant.
- Next time . . .

Minute Essay

Slide 8

- Questions?
- Have you taken Discrete (CSCI 1323)? With whom? If it's not nosy — did you like it? did you do well?