





A Little About Random Numbers
(Sources: Knuth, Quinn, SPRNG Web site.)
Many application areas that depend on "random" numbers (whatever we mean by that) — simulation (of physical phenomena), sampling, numerical analysis (Monte Carlo methods, e.g.), programming (to generate data, also some algorithms), etc.
Early on, people used physical methods (currently still in use in lotteries), and thought about building hardware to generate "random" results. No good large-scale solution, though, and besides it seemed useful to be able to repeat a calculation.
Hence need for "random number generator" (RNG) — way to generate "random" sequences of elements from a given set (e.g., integers or doubles). Tricky topic. Many early researchers got it wrong. Many application writers aren't interested in details.

Desirable Properties of RNG --- "Randomness"

 Obviously a key goal, if tricky to define. A thought-experiment definition: Suppose we're generating integers in the range from 1 through *d*, and we let an observer examine as much of the sequence as desired, and ask for a guess for any other element in the sequence. If the probability of the guess being right is more than 1/*d*, the sequence isn't random.

Slide 5

- Also want uniformity for each element, equal probability of getting any of the possible values.
- For some applications, also need to consider "uniformity in higher dimensions": Consider treating sequence as sequence of points in 2D, 3D, etc., space. Are the points spread out evenly?

Other Desirable Properties of RNG

 Reproducibility. For some applications, not important, or even bad. But for many others, good to be able to repeat an experiment. Usually meet this need with "pseudo random number generator" — algorithm that computes sequence using initial value (seed) and definition of each element in terms of previous element(s).

- Speed. Probably not a major goal, though, since most applications involve lots of other calculations.
- Large cycle length. If every element depends only on the one before, once you get the initial element again what happens? and usually that's not good.



Approaches to Parallelizing RNGs • Central server — use one UE to generate sequence, have it distribute results to other UEs or let them request them. Reproducible? Efficient? Other problems? • Cycle division - split elements of original sequence between UEs, having each UE generate "its" elements. Two basic schemes - "leapfrog" and "cycle splitting". Reproducible? Efficient? Other problems? • Parameterization - e.g., "cycle parameterization" exploits property that some RNGs can generate different cycles depending on seed. Idea is to "parameterize" algorithm so UEs generate different cycles. Reproducible? Efficient? Other problems?

• Linear Congruential Generator (LCG). $x_n = (ax_{n-1} + b) \mod m$ m constrains cycle length (period) — usually prime or a power of 2. a and c

m constrains cycle length (period) — usually prime or a power of 2. a and c must be carefully chosen. Results good overall, but least significant bits "aren't very random", which affects how well they work for generating points in 2D, etc., space.

• Lagged-Fibonacci Generator.

 $x_n = (x_{n-j}opx_{n-k}) \mod 2^m, \quad j < k$

where op is + (additive LFG) or × (multiplicative LFG). Again, k must be carefully chosen. Must also choose "enough" initial elements.





Parallel RNG With Shared Memory

- Thread safety an issue, but have access to shared state, which might be attractive.
- Adaptation of "central server" idea use regular library function, but ensure one-at-a-time access — seems attractive. For us, might be effective, especially if we generate elements two at a time. Efficient?
- Other approaches similar to distributed-memory case, but require that each thread have its own "internal state". Could be a problem if using library functions.



