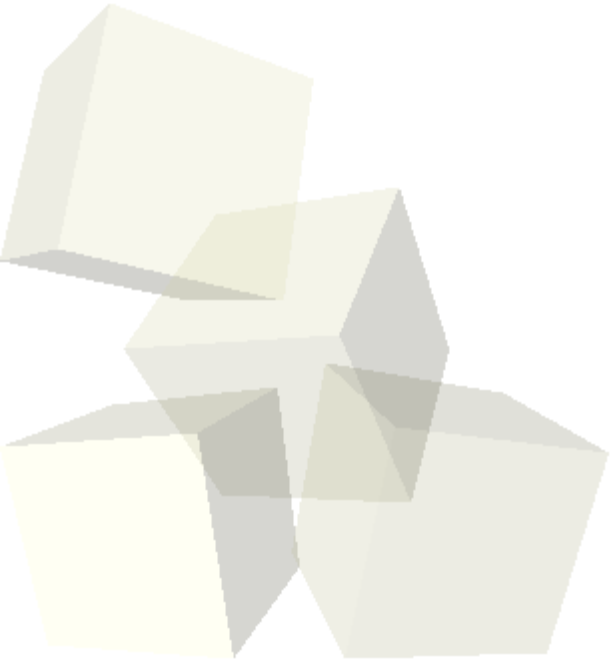
Symplectic Integrators and N-body

3/3/2008

Opening Discussion

- What did we talk about last class?



Other N-body/Multibody Systems

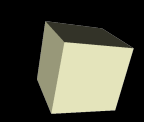
- Other common N-body type systems include collisional systems, molecular dynamics, granular flows, etc.
- Collisions can be handled through either hard or soft sphere means. Hard sphere doesn't work with an integrator, but soft sphere does, assuming the integrator is advanced enough.
- Boundary conditions can also complicate things. These are reasons why a large system likely wouldn't be integrated with something like ode45.

Symplectic Integrators

- You have seen that ode45 fails to do a good job of conserving energy in the systems we have given it. We could try to increase the accuracy, but that's just a stop-gap. We really need a different type of integrator.
- To understand symplectic integrators we should talk briefly about Hamiltonian systems. They are defined by a value H(p,q) which is basically the total energy of the system in terms of momenta (p) and positions (q) of the bodies.
- For Hook's law $H=0.5*p^2/m+0.5*kq^2$. This is just kinetic plus potential energy. The time derivatives of p and q are given by the partial derivatives of H.

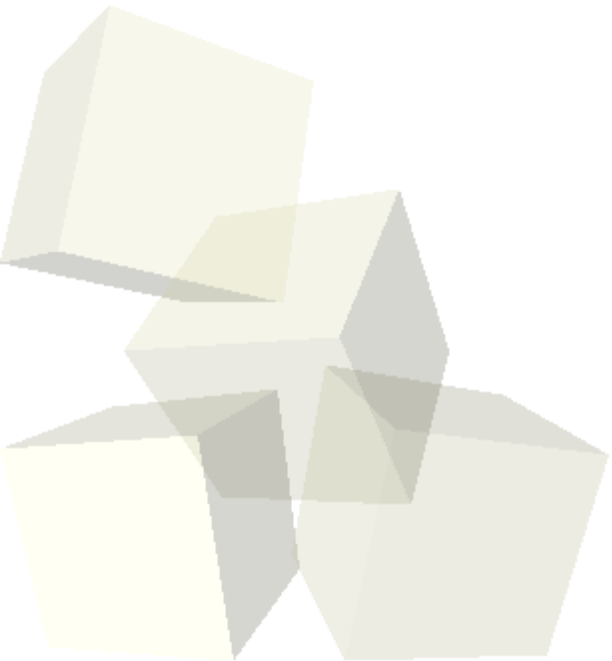
The Leapfrog

- We can build a symplectic integrator by breaking the problem into pieces that can be solved exactly, then alternating between solving those pieces (this is a simplified view). We will use a T+V style integrator also known as a kick-step method.
- Given the current position we calculate change in momentum and apply that. Then we take a step using the new momentum. Simply repeat this for the integration.
- This will perfectly integrate some Hamiltonian system that is similar to the one we are really interested in.



Project

- So what ideas have you come up with for your project?



Closing Comments

- Assignment #5 is due today.
- Remember that the midterm is Wednesday of next week.