Probability and Statistics

2/2/2009

Opening Discussion

- What did we talk about last class?

Random Variables

- This is a sequence of random numbers.
- Discrete
  - The values can only take on a countable number of possibilities.
- Continuous
  - The value comes from a non-countable set.

Differential Distribution Functions

- Your book calls these probability distribution functions.
- Discrete
  - $p(x_i) = P(X = x_i)$
  - $\sum_{i=1}^{\infty} p(x_i) = 1$
- Continuous
  - $P(X \in [x, x+\Delta x]) = \int_{x}^{x+\Delta x} f(y)\,dy$
  - $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Cumulative Distribution Functions

- Discrete
  - $F(x)=\sum_{x_i \le x} p(x_i)$
- Continuous
  - F(x)=P(X<=x) or
  - $F(x)=\int_{-\infty}^{x} f(y)\,dy$

Mean or Expected Value

- Discrete
  - $E(X_i) = \mu_i = \sum_{j=1}^{\infty} x_j \, p_{X_i}(x_j)$
- Continuous
  - $E(X_i) = \mu_i = \int_{-\infty}^{\infty} x f_{X_i}(x) \, dx$
- Properties
  - $E(cX) = cE(X)$
  - $E(\sum_{i=1}^{n} c_i X_i) = \sum_{i=1}^{n} c_i E(X_i)$

Median

- Discrete
  - Smallest x such that $F(x)>=0.5$
- Continuous
  - $F(x)=0.5$

Variance

- $\sigma_i^2 = E[(X_i - \mu_i)^2] = E(X_i^2) - \mu_i^2$
- Standard deviation is square root of variance.
- Properties

$$Var(X) \geq 0$$

$$Var(cX) = c^2 Var(X)$$

$$Var(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} Var(X_i)$$

Covariance

- $C_{ij} = E[(X_i - \mu_i)(X_j - \mu_j)] = E(X_i X_j) - \mu_i \mu_j$
- Measures correlation between random variables.
- Independent random variables have $C_{ij}=0$.
- Sign tells you if they are positively of negatively correlated
- Correlation ρ_{ij} fixes units problem.

Stochastic Processes

- This is a collection of similar random variables ordered over time. Our simulation outputs can be considered to be stochastic processes.
- Covariance-stationary implies that the mean and variance don't change over time.

Minute Essay

- Do you have any questions about what we covered today?