

# Random Number Generators

3/2/2009

# Opening Discussion

- Midterm results.

# Random Number Generators

- For obvious reasons we need good random numbers for our simulations.
- Text has some interesting history on getting random numbers, much of it prior to the use of computers.
- Use of the term random number isn't quite right. Author doesn't like pseudo-random.
- Really care about the properties of the sequence.

# Properties of Good Random

- There are several properties we want from a random number generator. We are always talking about something that generates  $U(0,1)$ .
  - Uniformly distributed and apparently uncorrelated.
  - Fast and small storage.
  - Reproducible.
  - Have ability to pull from separate streams.
  - Portable.

# Linear Congruential Generators

$$Z_i = (aZ_{i-1} + c) \pmod{m}$$

$$U_i = Z_i / m$$

$$0 < m, a < m, c < m, Z_0 < m$$

- Will eventually repeat. Length is called period. Optimally it takes  $m$  steps, but won't in all cases.

# Requirements

- A LCG will have full period if:
  - $m$  and  $c$  are relatively prime (only common divisor is 1)
  - If  $q$  is a prime number that divides  $m$ , then  $q$  divides  $a-1$ .
  - If 4 divides  $m$ , then 4 divides  $a-1$ .
- Mixed generators set  $m=2^b$ ,  $c$  is odd, and  $a$  is divisible for 4. Make  $b$  the size of numbers on the machine.

# Other Generators

- General Congruence generators
  - $Z_i = g(Z_{i-1}, Z_{i-2}, \dots) \pmod{m}$
- Composite Generators combine results from 2+ generators.
  - Shufflers use output of one as input to another.
  - Can use difference of two sequences.
- Feedback shift generators
  - Operate on the bits of a number.

# Minute Essay

- ???