More Generation

3-9-2011

Opening Discussion

- Minute essay comments:
  - MAS for the project.
  - Proper scope of the projects.
  - Complex projects and safety net.

Convolution

- If $X=Y_1+Y_2+...+Y_n$ and we can generate the Y values we simply do so.
- This is different from composition where the F functions are summed.

Acceptance-Rejection

- This method is somewhat indirect. We generate values and reject them if they aren't good.
- Pick majorizing function $t(x) >= f(x)$ for all x. Let $r(x)$ be $t(x)/c$ where c is the integral over t.
- Process
  - Generate Y from r.
  - Generate U independent of Y.
  - If $U <= f(Y)/t(Y)$, return $X=Y$. Otherwise goto 1.

Graphical Interpretation

- Let's look at a graphical description of what acceptance-rejection is doing.
- Note that this works best if c is close to one because we only accept 1/c of the generated values.
- It is easy to use a constant t(x), but that isn't always efficient.

Ratio of Uniforms

- I'll let you read about this one in the book. I'm not even going to try doing ti in class.

Special Properties

- Some distributions have other nice properties that we can use to help with generating them.
- Often this is a mathematical relationship to some other distribution.
- You can view convolution as a type of special property.

Generating Continuous Variates

- Now that we know of several ways to generate continuous random variates we should apply them to the different distributions we looked at back in chapter 6.

Uniform

- $U(a,b)=a+(b-a)U(0,1)$

Exponential

- $X = -\beta \ln U$

m-Erlang

- Generate U_1, U_2, ..., U_m
- $X=-\beta/m \ln(U_1 * U_2 * \ldots * U_m)$

Gamma

- There are several methods for generating gamma distributions. Because we can't get $F^{-1}(u)$ these are generally acceptance-rejection methods.
- Generate U_1 and U_2.
- $V=a \ln[U_1/(1-U_1)]$, $Y=\alpha e^V$, $Z=U_1^2U_2$, $W=b+qV-Y$.
- If $W+d-\theta Z>=0$ return $X=Y$
- If $W>=\ln Z$ return $X=Y$/ Otherwise start at beginning.

Weibull

- $X=\beta(-\ln U)^{1/\alpha}$

Normal

- Generate U_1 and U_2. $V_i=2U_i-1$, $W=V_1^2+V_2^2$.
- If W>2 return to first step. Otherwise, $Y=sqrt((-2lnW)/W)$, $X_1=V_1Y$, $X_2=V_2Y$.

Lognormal

- $Y \sim N(\mu, \sigma^2)$
- $X = e^Y$

Beta

- Y_1~gamma(α_1,1), Y_2=gamma(α_2,1)
- $X=Y_1/(Y_1+Y_2)$

Empirical

- How you do this depends on the type of empirical distribution.
- The book presents methods that don't require doing a search through an array.

Minute Essay

- Questions?
- What are your plans for Spring Break?