Computational Geometry

4-27-2011

Opening Discussion

- Do you have any questions?

- Minute Essay comments

  - How do Monte Carlo simulations differ from other simulations that use lots of random numbers?

  - Do there have to be a discrete number of states in a Markov chain?

Writing a Markov Chain

- Keep an array (possibly with multiple dimensions) for the states.

- Edges can be represented as lists or a matrix, choice depends on whether they are dense of sparse.

- Alternately, edges could be a function if coefficients depend on state.

Computational Geometry

- Many different types of problems require handling geometry.

- When simulations are done with a spatial element they also include geometric elements.

- Computational geometry is the study of efficient and correct algorithms for dealing with geometry.

Convex Hull

- One example is finding the convex hull of a set of points. This is the smallest convex polygon that contains all the points.

- A shape is convex if given any two points, a and b, in the shape, all the points on segment ab are also in the shape.

First Algorithm

- Run through all pairs of points
  - For each pair if all other points are to the right of the directed line, add that segment to a list.
- Link up the segments that you find in the end.
- This has several problems.
  - It isn't robust with floating point numbers.
  - It needs to be adjusted for degeneracies.
  - It is $O(n^3)$.

Improved Algorithm

- Sort the points by x and add first two to a list.

- Run through remaining points and

  - Append next point to list

  - While the list has more than two points and the last three don't make a right turn

    - Delete the middle of the last three.

- Repeat this process in reverse order to make lower hull.

- Append lower and upper.

- Runs in O(n log n) time.

Spatial Partitioning

- When we talked about collisions we talked about using a grid to partition the space so that we could find collisions efficiently.

- Grids are fast, but they are not very flexible. Trees are much more flexible.

- The 1-D example of a tree is something you will find familiar.

- Data can go in all nodes or just leaves.

Fast Gravity Calculations

- One use of spatial partitioning in simulation has been efficient approximations to gravitational forces.

- Like collisions, a standard gravity algorithm requires $O(n^2)$ work. Unlike collisions, gravity is long range so you can't just search nearby.

- Long range forces can be approximated by grouping particles. Spatial trees are the standard method of doing this.

Minute Essay

- Do you have any questions?
- You should turn in your test report by tonight.