Multibody Systems

2-29-2010

Opening Discussion

- What did we talk about last class?
- Do you have any questions about the assignment?

- Last time we started working on a function to do a system of mutually attracting massive bodies.
- Let's finish that up and see if we can finish this N-body integrator.

Real Gravity Simulations

- Real simulations of gravitational systems would never be done this way. The lack of energy conservation is a serious problem for long term integrations.
- Small systems have to go a long time normally so a symplectic integrator would be used.
- Large systems would have problems with the $O(n^2)$ nature of what we have written. Tree codes can improve this to O(n log n). Multipole methods can run in O(n) time. The coefficients and complexity go up with each of these.

Other N-body/Multibody Systems

- Other common N-body type systems include collisional systems, molecular dynamics, granular flows, etc.
- Collisions can be handled through either hard or soft sphere means. Hard sphere doesn't work with an integrator, but soft sphere does, assuming the integrator is advanced enough.
- Boundary conditions can also complicate things. These are reasons why a large system likely wouldn't be integrated with something like ode45.

Symplectic Integrators

- You have seen that ode45 fails to do a good job of conserving energy in the systems we have given it. We could try to increase the accuracy, but that's just a stop-gap. We really need a different type of integrator.
- To understand symplectic integrators we should talk briefly about Hamiltonian systems. They are defined by a value H(p,q) which is basically the total energy of the system in terms of momenta (p) and positions (q) of the bodies.
- For Hook's law $H=0.5*p^2/m+0.5*kq^2$. This is just kinetic plus potential energy. The time derivatives of p and q are given by the partial derivatives of H.

The Leapfrog

- We can build a symplectic integrator by breaking the problem into pieces that can be solved exactly, then alternating between solving those pieces (this is a simplified view). We will use a T+V style integrator also known as a kick-step method.
- Given the current position we calculate change in momentum and apply that. Then we take a step using the new momentum. Simply repeat this for the integration.
- This will perfectly integrate some Hamiltonian system that is similar to the one we are really interested in.

Reminders

- Assignment #5 is due today.
- No class on Wednesday because of jury duty.