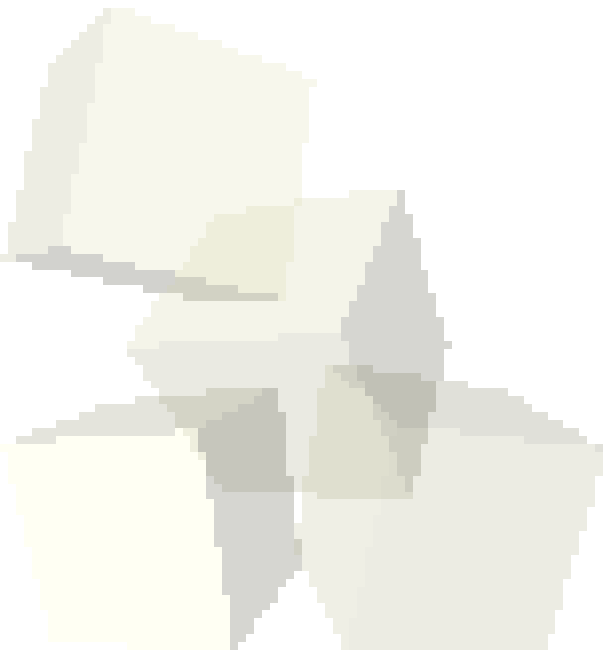




Multibody Systems

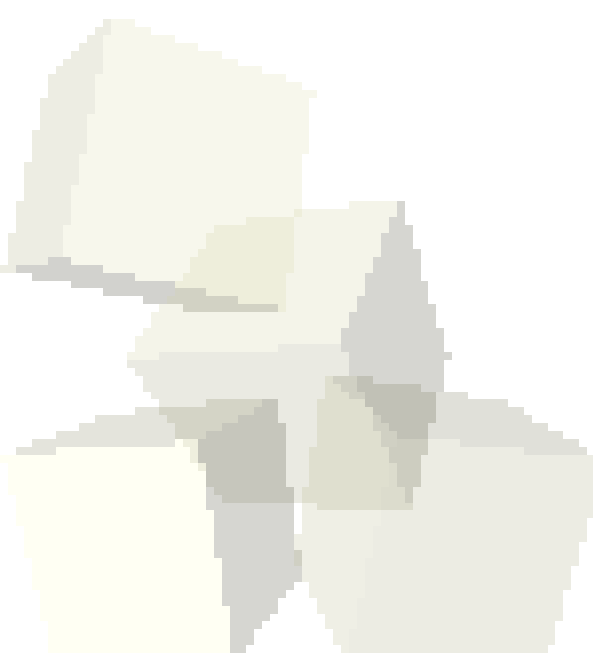
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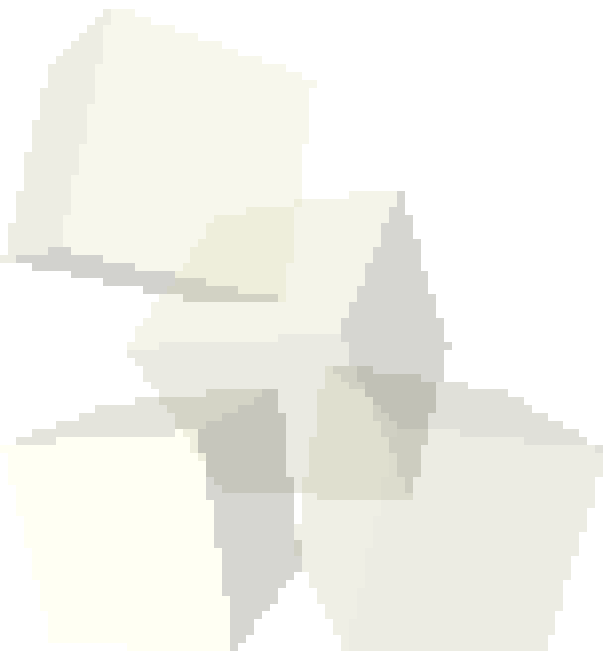
Opening Discussion

- What did we talk about last class?
- Do you have any questions about the assignment?





- Last time we started working on a function to do a system of mutually attracting massive bodies.
- Let's finish that up and see if we can finish this N-body integrator.





Real Gravity Simulations

- Real simulations of gravitational systems would never be done this way. The lack of energy conservation is a serious problem for long term integrations.
- Small systems have to go a long time normally so a symplectic integrator would be used.
- Large systems would have problems with the $O(n^2)$ nature of what we have written. Tree codes can improve this to $O(n \log n)$. Multipole methods can run in $O(n)$ time. The coefficients and complexity go up with each of these.



Other N-body/Multibody Systems

- Other common N-body type systems include collisional systems, molecular dynamics, granular flows, etc.
- Collisions can be handled through either hard or soft sphere means. Hard sphere doesn't work with an integrator, but soft sphere does, assuming the integrator is advanced enough.
- Boundary conditions can also complicate things. These are reasons why a large system likely wouldn't be integrated with something like ode45.



Symplectic Integrators

- You have seen that ode45 fails to do a good job of conserving energy in the systems we have given it. We could try to increase the accuracy, but that's just a stop-gap. We really need a different type of integrator.
- To understand symplectic integrators we should talk briefly about Hamiltonian systems. They are defined by a value $H(p,q)$ which is basically the total energy of the system in terms of momenta (p) and positions (q) of the bodies.
- For Hook's law $H=0.5*p^2/m+0.5*kq^2$. This is just kinetic plus potential energy. The time derivatives of p and q are given by the partial derivatives of H .



- We can build a symplectic integrator by breaking the problem into pieces that can be solved exactly, then alternating between solving those pieces (this is a simplified view). We will use a T+V style integrator also known as a kick-step method.
- Given the current position we calculate change in momentum and apply that. Then we take a step using the new momentum. Simply repeat this for the integration.
- This will perfectly integrate some Hamiltonian system that is similar to the one we are really interested in.



- Assignment #5 is due today.
- No class on Wednesday because of jury duty.

