

Probability Distributions

2/16/2009

Opening Discussion

- What did we talk about last class?
- Did anyone take the time to play with the things that I posted late on Friday?

Need for Distributions

- Many processes that we might want to simulate involve processes that include some randomness.
- We have already seen that just using a mean value is bad.
- Using the wrong distribution can also mess things up.

Sources of Random Numbers

- Actual Data
 - Good for verification.
 - Limited for real runs.
- Empirical Distribution
 - Build a distribution from the data.
 - Artificially bounded.
- Theoretical Distribution from fit
 - Ideal solution if good distribution can be found.

Continuous Distributions

- The book goes through a bunch of different continuous distributions including lots of details and plots.
 - $U(a,b)$ – Uniform
 - Use when you don't know any better.
 - $\text{expo}(\beta)$ - Exponential
 - Inter-arrival times or failure times.
 - $\text{gamma}(\alpha,\beta)$
 - Task completion

More Continuous Distributions

- Weibull(α, β)
 - Task completion of equipment failure. Rough model in absence of data.
- $N(\mu, \sigma^2)$ - Normal
 - Errors or sum of many values.
- $LN(\mu, \sigma^2)$ – Lognormal
 - Task completion with long tail.
- $\text{beta}(\alpha_1, \alpha_2)$
 - Rough model or distribution of random proportions.

More Continuous Distributions

- $PT5(\alpha, \beta)$ – Pearson type V
 - Time to perform task.
- $PT6(\alpha, \beta)$ – Pearson type VI
 - Time to perform task.
- $LL(\alpha, \beta)$ – Log-logistic
 - Time to perform task
- $JSB(\alpha_1, \alpha_2, a, b)$ - Johnson S_B
- $JSU(\alpha_1, \alpha_2, \gamma, \beta)$ - Johnson S_U
- $triang(a, b, m)$

Discrete Distributions

- There are also established distributions for discrete values.
 - Bernoulli(p)
 - Coin flip where odds aren't always equal.
 - $DU(i,j)$ – Discrete Uniform
 - Several outcomes of equal probability. First cut.
 - $\text{bin}(t,p)$ – Binomial
 - Number of successes in t Bernoulli trials.
 - $\text{geom}(p)$ – Geometric
 - Number of tries before a fail.

More Discrete Distributions

- $\text{negbin}(s,p)$ – Negative Binomial
 - Number of failures before the s^{th} success.
- $\text{Poisson}(\lambda)$
 - Number of items demanded from inventory.

Empirical Distributions

- You can build your own distributions from empirical data.
- If data isn't binned, sort it.
 - $F(x) = \text{indexOf}(x)/n$ if x is data point. Otherwise interpolate.
- If data is binned you can build $F(x)$ from the binned data in a similar way.
- Both have the downside that values have a limited range.

Minute Essay

- Why do you think that there are so many of these different distributions?