Continuous Systems

2-11-2011

Opening Discussion

- Do you have any questions about the quiz?
- Do you have any questions about the reading or the assignment?
- When you can't observe the system or consult an SME, how should you get good data?
- Minute essays
  - Traffic sensors: http://ask.metafilter.com/69006/Those-black-traffic-cable
  - Challenges of credibility.
  - Frameworks for animation.

Continuous Systems

- Unlike the discrete event systems we have been talking about, these systems change in a continuous way over time.
- Most of the time they are modeled with differential equations. Simulation is used because only the simplest of differential equations can actually be solved by hand.
- The physical world is pretty much a continuous system. Why then do we do discrete simulations?

Ordinary Differential Equations

- ODEs involve one or more functions of a single variable and the derivatives of those functions.
- This is opposed to partial differential equations which involve functions of multiple variables and their partial derivatives.
- Higher order ODEs can be converted to systems of first order ODEs.

Basic ODE Solvers

- $\frac{y}{dt} = y' = f(t, y(t)), t_{n+1} = t_n + h$
- Euler method
  - $y_{n+1} = y_n + hf(t_n, y_n)$
- Runga-Kutta methods (this is 4th order)
  - $$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$$
    $$k_1 = y(t_n, y_n)$$
    $$k_2 = y(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1)$$
    $$k_3 = y(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2)$$
    $$k_4 = y(t_n + h, y_n + hk_3)$$

Hamiltonian Dynamics

- This is an alternate description of Newtonian mechanics for conservative systems. It can also work for other types of systems.
- The Hamiltonian, H(**p**,**q**), is the total energy in a conservative system.

$$\dot{p}_i = \frac{-\partial H}{\partial q_i}$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

Symplectic Integrators

- These are specialty integrators for Hamiltonian systems.
- They do an exact solution to a Hamiltonian close to the one we are actually trying to integrate. As long as the two Hamiltonians are sufficiently close this provides energy conservation, preserves volumes in phase space and other Poincaré invariants.

Minute Essay

- Do you have any questions about what we talked about today?
- Remember that assignment #2 is due on Monday.