

Derivative m D. n u D. n mu

$u D. n$ is the n -th derivative of u , and $u`v D. n$ is u with an assigned n -th derivative v . For example:

```
(cube D.1;cube D.2;(cube=: ^&3"0) D.3)y=: 2 3 4
+-----+-----+-----+
|12 27 48|12 18 24|6 6 6|
+-----+-----+-----+
```

The derivative applies to constant functions, polynomials, the exponential e^x , the integral powers x^n , and those assigned by $u`v D. n$. It also applies to functions derived from these by addition, subtraction, multiplication, and division ($u+v$, etc.); by the composition $u@v$; and by the inverse $u^:_1$. Since functions such as $\sqrt{\quad}$ and $-$ (negation) and $\%:$ (square root) and $1&o.$ (sin) and $6&o.$ (cosh) may all be so derived, they are also in the domain of the derivative. Others are treated by approximation. The derivative of an arbitrary function may also be treated by a polynomial approximation, (provided by the matrix divide), or by approximations using the secant slope $D:$.

If the argument rank of u is a and the result rank is r , then the argument rank of $u D.1$ is also a , but its result rank is $r+a$: the result of $u D.1$ is the derivative of each atom of the result of u with respect to each element of its argument, giving what is commonly referred to as the partial derivatives. For example:

```
volume=: */"1 [. VOLUMES=: */\"1
(volume;volume D.1;VOLUMES;VOLUMES D.1) y
+-----+-----+-----+
|24|12 8 6|2 6 24|1 3 12|
|0 2 8|
|0 0 6|
+-----+-----+-----+
determinant=: -/ . * [. permanent=: +/ . *
( );(determinant D.1);(permanent D.1)m=:*:i.3 3
+-----+-----+-----+
|0 1 4|_201 324 _135|2249 1476 1017|
|9 16 25|_132 _144 36|260 144 36|
|36 49 64|_39 36 _9|89 36 9|
+-----+-----+-----+
```

The adverbs $D=: ("0)(D.1)$ and $VD=: ("1)(D.1)$ assign ranks to their arguments, then take the first derivative; they are convenient for use in scalar and vector calculus:

```
sin=: 1&o. [. x=: 0.5p1 _0.25p1
(*/\ VD y);(sin x);(sin D x);(sin D D x)
+-----+-----+-----+
|1 3 12|1 _0.707107|0 0.707107|_1 0.707107|
|0 2 8|
|0 0 6|
+-----+-----+-----+
```