CSCI 1323 (Discrete Structures), Spring 2006

Quiz 4 Solution

1. (6 points) A chat program requires each user to have a nickname consisting only of letters (lower case only) and digits (0 through 9), with the first character a letter. Nicknames must be at least four characters long and cannot be longer than eight characters. (For example, `me00` and `myname` are allowed; `me`, `00me`, and `me11me11me11me11` are not.) How many people can use this system? I.e., how many distinct nicknames are possible? (It's fine to write your answer in the form of an arithmetic expression, e.g., $26^2 + 4$.)

   Solution: $(26 \times 36^3) + (26 \times 36^4) + (26 \times 36^5) + (26 \times 36^6) + (26 \times 36^7)$

   Would your answer be different if upper case letters were allowed, but case was not significant (i.e., `me00` and `ME00` are "the same")? (It's okay to just answer "yes" or "no".)

   Solution: No.

2. (4 points) In class we noted that there are many sets that are *countably infinite* ("the same size as" the set of positive integers) — the integers, the even integers, and even the rational numbers. Is the set of prime numbers also countably infinite? Why or why not? (To be countably infinite, it has to be both infinite and countable.)

   Solution: Yes — it's infinite because there is no largest prime number, and it's countable because we can list the primes in ascending order.