

Minute Essay From Last Lecture • Most people had not seen BSTs in CS2 — yet? — but many had in Functional Languages. If you haven't encountered them, or don't feel fairly comfortable with them at a picture-drawing level, let me know and I'll (try to?) help.







## Machine Arithmetic — Integer Addition and Negative Numbers

- Adding binary numbers works just like adding base-10 numbers work from right to left, carry as needed. (Example.)
- Two's complement representation of negative numbers is chosen so that we easily get 0 when we add -n and n.
  - Computing -n is easy with a simple trick: If m is the number of bits we're using, addition is in effect modulo  $2^m$ . So -n is equivalent to  $2^m n$ , which we can compute as  $((2^m 1) n) + 1)$ .
- So now we can easily (?) do subtraction too to compute a b, compute -b and add.



## **Computer Representation of Real Numbers**

- How are non-integer numbers represented? usually as floating point.
- Idea is similar to scientific notation represent number as a binary fraction multiplied by a power of 2:

Slide 8

$$x = (-1)^{sign} \times (1 + frac) \times 2^{bias + exp}$$

and then store  $sign \ frac$ , and exp. Sign is one bit; number of bits for the other two fields varies — e.g., for usual single-precision, 8 bits for exponent and 23 for fraction. Bias is chosen to allow roughly equal numbers of positive and negative exponents.

• Current most common format — "IEEE 754".



- The integers and real numbers of the idealized world of math have some properties not completely shared by their computer representations.
- Math integers can be any size; computer integers can't.
- Math real numbers can be any size and precision; floating-point numbers can't. Also, some quantities that can be represented easily in decimal can't be represented in binary.
- Math operations on integers and reals have properties such as associativity that don't necessarily hold for the computer representations. (Yes, really!)
- (Two "floating point is strange" examples.)



