Slide 1

Administrivia

- ACM meeting Thursday at 4pm in HAS 329. Find out what this organization is/does. Free refreshments!

Slide 2

Minute Essay From Last Lecture

- Question:
  - Consider formulas $Q(a)$, $Q(b)$, $(\forall x)Q(x)$. Tell me whether each is true or false for the following interpretations.
  - Interpretation 1: domain of interpretation is the integers, $a = 1$, $b = 2$, and $Q(x)$ means "$2x$ is an even integer".
  - Interpretation 2: domain of interpretation is the rational numbers, $a = 1/2$, $b = 1$, and $Q(x)$ means "$2x$ is an even integer".
- Answers?

Slide 3

Propositional Logic Versus Predicate Logic

- In propositional logic:
  - Wffs are true or false, depending on assignment of truth values to statement letters.
  - If a wff is true for all such assignments, "tautology" — always true.
  - Can show this by checking all cases (truth table).
- In predicate logic:
  - Wffs are true or false (or neither, if they have free variables), depending on "interpretation" (domain plus meanings for predicates and constants).
  - If a wff is true for all such interpretations, "valid" — always true.
  - *Cannot* show this by checking all cases.

Slide 4

Valid Arguments, Revisited

- As with propositional logic, we want to know when we can say that a conclusion "logically follows" from a set of hypotheses — i.e., no matter what interpretation we choose, if the hypotheses are true so is the conclusion.
- What we have in our "bag of tricks":
  - All propositional-logic rules.
  - New rules for manipulating quantifiers.

Universal Instantiation

- Rule for removing $\forall$. (Why do we want to do this?)
- If we have $(\forall x)P(x)$
  we can write $P(t)$
  provided t doesn't already exist "bound" in $P(x)$.
- "If $P(x)$ for all x, then $P(t)$ for a particular t".

Slide 5

Existential Instantiation

- Rule for removing $\exists$. (Why do we want to do this?)
- If we have $(\exists x)P(x)$
  we can write $P(t)$
  provided t has not been previously used in the proof.
- "If there is some x for which $P(x)$, we can give it a name — t, for example."

Slide 6

Universal Generalization

Slide 7

- Rule for introducing $\forall$. (Why do we want to do this?)
- If we have $P(x)$
  we can write $(\forall x)P(x)$
  provided x is "arbitrary" — not a free variable in a hypothesis, not a variable we got from ei.
- "If we know $P(x)$ for arbitrary x, then $P(x)$ for all x."

Existential Generalization

Slide 8

- Rule for introducing $\exists$. (Why do we want to do this?)
- If we have $P(y)$ or $P(a)$
  we can write $(\exists x)P(x)$
  provided x doesn't appear in $P(a)$.
- "If we have some particular z for which $P(z)$, then there exists a z such that $P(z)$."

Minute Essay

- None — quiz.

Slide 9