## Administrivia

- None.


## Slide 1

## Infinite Sets are Interesting, Continued

- Finite sets have finite sizes.
- The "smallest" infinite set is $\mathbb{N}$. Many other infinite sets are the "same size" e.g., $\mathbb{Z}, \mathbb{Q}$. This set's size is referred to as $\aleph_{0}$ ("aleph null"). Assuming that these sizes can be ordered, the "next bigger" size is $\aleph_{1}$, etc.

Slide 2

- $\mathbb{R}$ is "bigger", and the Continuum Hypothesis says it has size $\aleph_{1}$. Interestingly enough, $\mathbb{R}^{2}$ is "the same size" as $\mathbb{R}$, etc.
- And then there are infinitely many bigger sizes, since in general we can prove that $S$ and $\mathscr{P}(S)$ are not "the same size". The proof is by contradiction and is - interesting? clever?


## Recursion Can Be Fun (?)

- Let's try to define integer arithmetic (well, for non-negative integers) without ints as follows:
- Let $n$ be some sort of list of $n$ elements. We could implement this as something even simpler than a linked list - just a chain of pointers.


## Slide 3

- Define "primitive" operations isZero, add1, sub1.
- Try to build arithmetic and relational operations using primitive operations and recursion.
- Do you think this is doable in actual code? How much slower do you think it will be?


## Minute Essay

- None - sign in.


## Slide 4

